MA 412 – Complex Variables
Exam #2

Name:

Instructions: To receive full credit you must show all work. Explain your answers fully and clearly. You may refer to theorems/facts in the book or from class. No calculators, books or notes of any form are allowed. Good luck!

Question	Score	Out of
1		18
2		12
3		16
4		10
5		10
6		10
7		24
Total		100

1. **(18 points)**

- Define what it means for a function $f(z)$ to be entire.

- Is the function $f(z) = e^z \sin(2z - 1)$ entire ? Explain your reasoning.

- Is the function $f(z) = \sqrt{z} = \exp(\frac{1}{2}Log(z))$ entire ? Explain your reasoning.

2. **(12 points)**
   Evaluate the following multivalued expressions

   - $\log(-2+2i)$

   - $(-i)^i$

3. **(16 points)**

Determine the region in which the following functions are analytic, carefully drawing the branch cuts and singularities. Explain your reasoning.

- $$\frac{Log(3-2z)}{z^2+16}$$

- $$\sqrt{z^2+25},$$

  where the principal branch of the square root is taken.

4. **(10 points)**

Compute the contour integral

$$\int_C \overline{z} dz$$

where C is the contour from $-3i$ to 3 along the circle $|z| = 3$ by parametrizing C and direct evaluation.

5. **(10 points)**

Evaluate the contour integral

$$\int_C \frac{dz}{\sqrt{z}}$$

where C is the contour from $z = 1 + i$ to $2 + 4i$ along the parabola $y = x^2$ and $\sqrt{z}$ denotes the principal branch. (Hint: find an antiderivative).

6. **(10 points)**
Show that

$$|\int_C \frac{z-1}{z^3+2}dz| \leq \frac{12}{25}\pi$$

where C is the part of the circle $|z| = 3$ from 3 to -3. Clearly show each step in your estimate and which inequalities are being used.

7. **(24 points)**
Let

$$f(z) = \frac{z^3}{(z+2)^2(z-4)}.$$

Evaluate the following contour integrals, in each case explaining your reasoning and referring to the relevant theorems.

(a) $\int_{C_1} f(z)dz$ where C_1 is the positively oriented circle $|z-i| = 1$

(b) $\int_{C_2} f(z)dz$ where C_2 is the positively oriented square with corners at $-3-i, -i, 2i, -3+2i$.

(c) $\int_{C_3} f(z)dz$ where C_3 is the negatively oriented circle $|z-5|=2$.

(d) $\int_{C_4} f(z)dz$ where C_4 is the positively oriented circle $|z|=8$. (Hint: how does this integral relate to those over C_2 and C_3 ?).