1. (3 points) Print first name: _____ and last name: _____

BU ID number (ex: U12345678) U_____

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD THAT YOU CAN BEGIN THE EXAM

Directions:

- The use of smart watches, calculators, entertainment devices, communication devices, and notes are not permitted during this exam.
- All phones and smart watches must be turned OFF (silent mode is not permitted), and they must be stored in your backpack. If you do not have a backpack, consult a proctor.
- All backpacks must be stored in the front of the room. Keep your BU ID with you.
- Do all of your work in this exam booklet and make sure that you erase or cross out any work that we should ignore when we grade.
- **Books and extra papers are not permitted. Do not separate the pages of this exam booklet.**
- If you have a question about a problem, raise your hand and a proctor will come to your seat to answer it.
- Answers that are written logically and clearly will receive higher scores.
- There are ten calculus problems and five logistical problems that simply ask for your name and/or ID number. The entire exam booklet consists of five double-sided pieces of paper including this cover page. Make sure that your exam booklet includes all five pieces of paper.

2. (20 points) In each part, compute dy/dx. Show your work but **do NOT simplify your answers.**

(a) $y = e^{-4x}\cos 2x$

(b) $y = \dfrac{\ln 3x}{x^2+5}$

(c) $y = e^{5x} + x^{3e} + \pi^2$

(d) $y = \tan^{-1}\left(\sqrt{3+\sin x}\right)$ (Recall that $\tan^{-1}$ is our notation for the inverse tangent function. It is often called the arctan function.)

3. (1/4 point) Print your BU ID number on this page too: U_____

4. (6 points) Calculate the indefinite integral $\int x^3 \sin x^4 \, dx$. Show enough work to justify your answer.

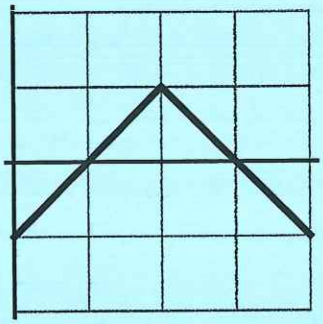
5. (12 points) Evaluate the following definite integrals. Show enough work to justify your answers, and simplify your answers by evaluating standard functions at known arguments whenever possible (for example, replace $\cos 0$ with its value 1, $\sin(\pi/3)$ by $\sqrt{3}/2$, etc.)

(a) $\displaystyle\int_1^{e^5} \frac{1}{x} \, dx$

(b) $\displaystyle\int_0^{1/2} \frac{3}{\sqrt{1-3x^2}} \, dx$

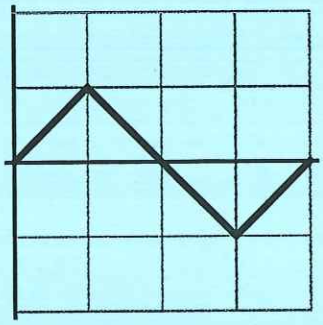
6. (12 points) For each of the following functions f that are graphed in parts (a)–(c), indicate which graph (1–9) is the graph of the net area function $N(x) = \int_0^x f(t)\,dt$. **Write the number for your answer in the corresponding answer box.** The scale is identical for all graphs. Each small square is a square with a width of 1 unit and a height of 1 unit. The horizontal axis is the interval $[0, 4]$, and the vertical axis is the interval $[-2, 2]$.

(a.)



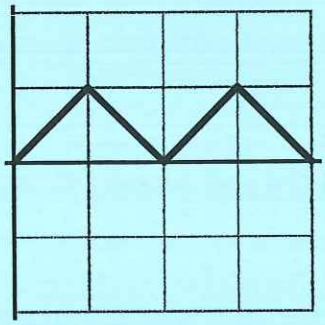
Answer: #_____

(b.)



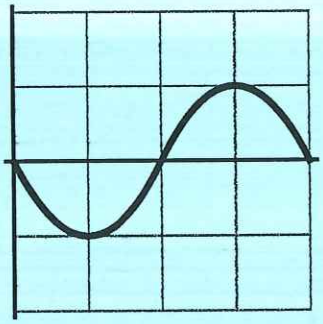
Answer: #_____

(c.)



Answer: #_____

(1.)



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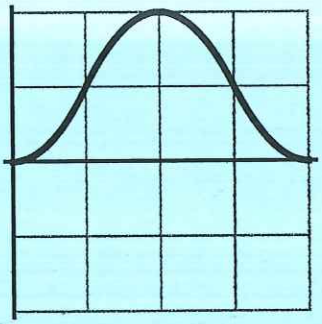

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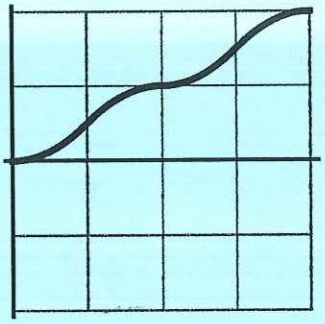

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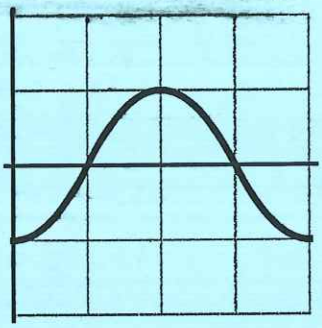

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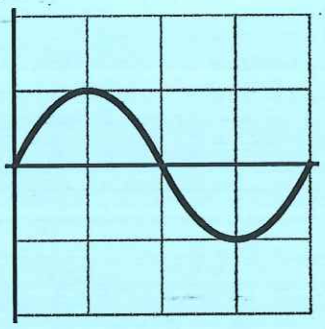

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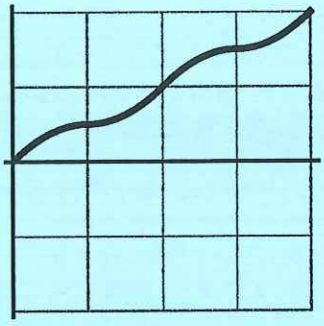
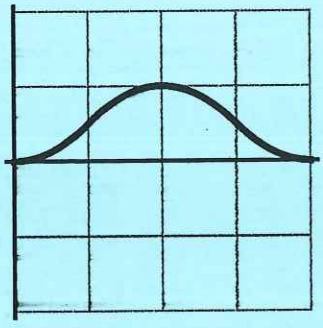


(7.)



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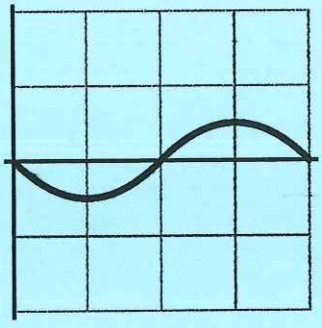

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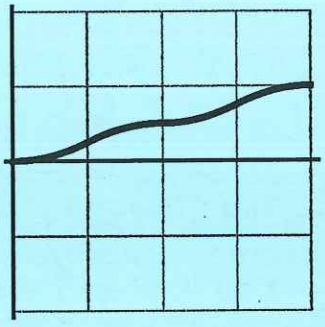
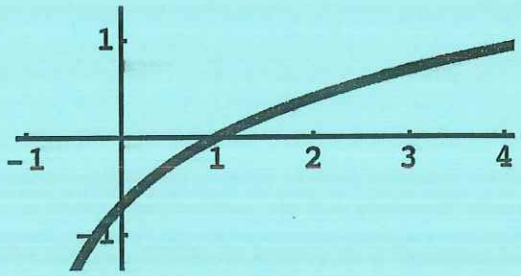


7. (1/4 point) Print your BU ID number on this page too: U_____

8. (4 points) Multiple choice: **Pick at most one answer**—the one that corresponds to the graph on the right. You indicate your choice by filling in the circle to the immediate left of your choice. You will not receive any credit for an ambiguous answer. You will receive 4 points for the correct answer, 1 point if you do not pick any of the choices, and 0 points for the wrong answer.

The graph of a twice-differentiable function f is shown to the right. Which of the following set of inequalities is true?



- ○ $f(1) < f''(1) < f'(1)$
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- ○ $f''(1) < f(1) < f'(1)$

9. (4 points) Let f be the function defined by

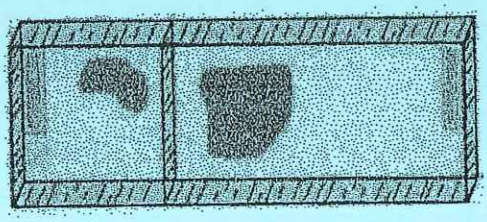
$$f(x) = \begin{cases} 4x - 7, & \text{if } x < 2; \\ 3, & \text{if } x = 2; \\ 3 - x, & \text{if } x > 2. \end{cases}$$

Many choice: Pick **all of the correct statement(s)** among the four statements below for the function f. You indicate that a statement is true by checking the box to the immediate left of the statement. **If the statement is false, leave the box blank.**

You will not receive any credit for ambiguous answers.

- ☐ The limit of $f(x)$ as $x \to 2^-$ exists.
- ☐ The limit of $f(x)$ as $x \to 2$ exists.
- ☐ The function f is continuous at $x = 2$.
- ☐ The function f is differentiable at $x = 2$.

10. (10 points) A farmer has 60 meters of fencing with which he plans to make a rectangular pig pen. The pen will have one internal fence that runs parallel to the end fences and divides the pen into two sections. Calculate the dimensions that produce the pen of maximum area assuming that the length of the larger section is to be twice the length of the of the smaller section (see the figure). Justify the fact that your answer corresponds to the pen of maximum area.



(a) Indicate the meaning of all variables that you will use to solve the problem.

(b) Derive the function of one variable that you will maximize to solve the problem (the objective function) and specify its domain.

(c) Find the desired dimensions that solve the problem using any of the techniques that have been discussed in this course.

(d) With one sentence or a brief calculation, explain why the dimensions that you produced in part (c) maximize the combined area.

11. (1/4 point) Print your BU ID number on this page too: U_____

12. (4 points) True or false:

$$\frac{d}{dx}\ln(4x) = \frac{d}{dx}\ln(x).$$

You indicate your choice by filling in the corresponding circle below:

○ True ○ False

You will not receive any credit for an ambiguous answer. You will receive 4 points for the correct answer, 1 point if you do not pick either choice, and 0 points for the wrong answer.

13. (18 points) Three limits: One on this page and two more on the next page. For each limit, evaluate it if it exists. If the limit is infinite, indicate if it is $+\infty$ or $-\infty$. If it does not exist and is not infinite, write DNE. Show enough work to justify your answers.

(a) $\lim_{x\to 2^+} \dfrac{x^2-9}{x^2+x-6}$

13. Problem 13 and its directions continued from the bottom of the previous page: For each limit, evaluate it if it exists. If the limit is infinite, indicate if it is $+\infty$ or $-\infty$. If it does not exist and is not infinite, write DNE. Show enough work to justify your answers.

(b) $\lim_{x\to 0}\left(e^{4x}+x\right)^{1/x}$

(c) $\lim_{x\to 1}\dfrac{\cos\left(\frac{\pi}{2}x\right)}{1-\sqrt{x}}$

14. (1/4 point) Print your BU ID number on this page too: U_____

15. (6 points) Consider the function $N(x) = \int_0^x \sqrt{25 - t^2}\, dt$. Its domain is the interval $-5 \leq x \leq 5$.

Calculate the three values $N(0)$, $N(5)$, and $N'(3)$ using geometric methods and/or theorems that have been discussed in this course. Show enough work to justify your answers and write the values in the boxes below. Each justification should be a brief calculation, a sentence, and/or a labeled diagram.

$N(0) =$ _____	$N(5) =$ _____	$N'(3) =$ _____

End of Exam

You may use the bottom of this page as well as the next page if you did not have enough space to answer a question, but **do not separate this page from the rest of the exam booklet.** If you want the graders to consider something that is written below this line or on the next page, make sure that there is a note to that effect next to the relevant problem. If you write something here that you would like the graders to ignore, simply cross it out.