1. (2 points) Print first name: ____________ and last name: ____________________

BU ID number (ex: U12345678) U__________

DO NOT OPEN THIS BOOKLET UNTIL YOU ARE TOLD THAT YOU CAN BEGIN THE EXAM

Directions:

- The use of smart watches, calculators, entertainment devices, communication devices, and notes are not permitted during this exam.
- All phones and smart watches must be turned OFF (silent mode is not permitted), and they must be stored in your backpack. If you do not have a backpack, consult a proctor.
- All backpacks must be stored in the front of the room. Keep your BU ID with you.
- Do all of your work in this exam booklet and make sure that you erase or cross out any work that we should ignore when we grade.
- **Books and extra papers are not permitted. Do not separate the pages of this exam booklet.**
- If you have a question about a problem, raise your hand and a proctor will come to your seat to answer it.
- Answers that are written logically and clearly will receive higher scores.
- There are 10 calculus problems and 5 logistical problems that simply ask for your name and/or ID number. The entire exam booklet consists of **five** double-sided pieces of paper including this cover page. Make sure that your exam booklet includes all **five** pieces of paper.

2. (20 points) In each part, compute dy/dx. Show your work but **you do not need to simplify your answers.**

(a) $y = x^4 \, e^{-3x}$

(b) $y = \dfrac{\cos 5x}{x^2 + 4}$

(c) $y = \sin^{-1}\left(\sqrt{3x}\right)$ (Recall that $\sin^{-1}$ is our notation for the inverse sine function. It is often called the arcsine function.)

(d) $y = \ln(e^{\tan 7x})$

3. (1/4 point) Print your BU ID number on this page too: U______________

4. (6 points) Assuming that $x > 0$, calculate the indefinite integral $\int \frac{3+\sqrt{x}}{x}\,dx$. Show enough work to justify your answer.

5. (6 points) Compute the slope of the line tangent to the curve $3x^2 - 2y^2 = 6 - 2xy$ at the point $(2, 3)$. Write the value of the slope in the box provided below and show enough work to justify your answer.

slope = ______________

6. (15 points) For each limit, evaluate it if it exists. If the limit is infinite, indicate if it is $+\infty$ or $-\infty$. If it does not exist and is not infinite, write DNE. Show enough work to justify your answers.

(a) $\lim_{x \to 0} \frac{x - \sin x}{5x^3}$

(b) $\lim_{x \to \infty} \tan^{-1}\left(1 + e^{-4x}\right)$

(c) $\lim_{x \to 0} \frac{e^{4x}}{5x}$

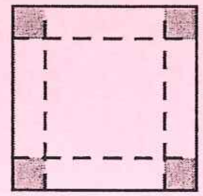
7. (1/4 point) Print your BU ID number on this page too: U_______________

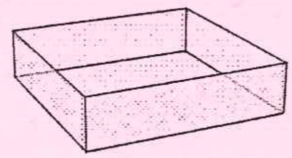
8. (12 points) Evaluate the following definite integrals. Show enough work to justify your answers, and simplify your answers by evaluating standard functions at known arguments whenever possible (for example, replace $\sin \pi/2$ with its value 1, e^0 by 1, etc.)

(a) $\displaystyle\int_0^{\pi/9} \cos 3x \, dx$

(b) $\displaystyle\int_0^1 \frac{x}{1+4x^2} \, dx$

9. (12 points) An open box (a box without a top) will be made from a square piece of cardboard that is 2 feet on a side by cutting equal-sized squares from each corner and folding up the sides as shown in the figures on the right. Answer the four questions below to determine the side length of the squares cut from each corner that maximizes the volume of the resulting box.





   (a) Indicate the meaning of any variables that you will use to solve the problem.

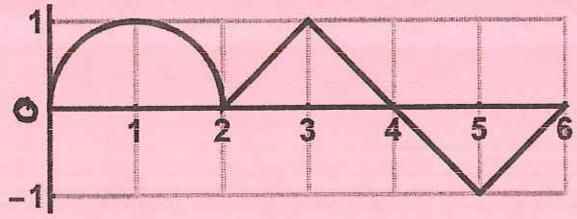
   (b) Produce a function of a single variable that you will maximize to solve the problem and specify its domain.

   (c) Find the desired dimension that solves the problem using any of the techniques that have been discussed in this course.

   (d) With one sentence or a brief calculation, explain why the dimension that you produced in part (c) maximizes the volume.

10. (1/4 point) Print your BU ID number on this page too: U______________

11. (4 points) Consider the function $f(t)$ that is graphed below. It is defined on the interval $[0, 6]$. (The curved arc is 1/2 of a circle.)



Let $G(x) = \int_0^x f(t)\, dt$ and $H(x) = \int_3^x f(t)\, dt$.

Calculate the two differences $G(3) - H(3)$ and $G(6) - H(6)$ and write your answers in the two boxes below:

(a) $G(3) - H(3) =$ __________

(b) $G(6) - H(6) =$ __________

12. (6 points) Suppose that the functions f and f' are continuous on an open interval I that contains the numbers a and b. Simplify the following expressions by eliminating the integral sign.

(a) $\int_a^b f'(x)\, dx =$ __________

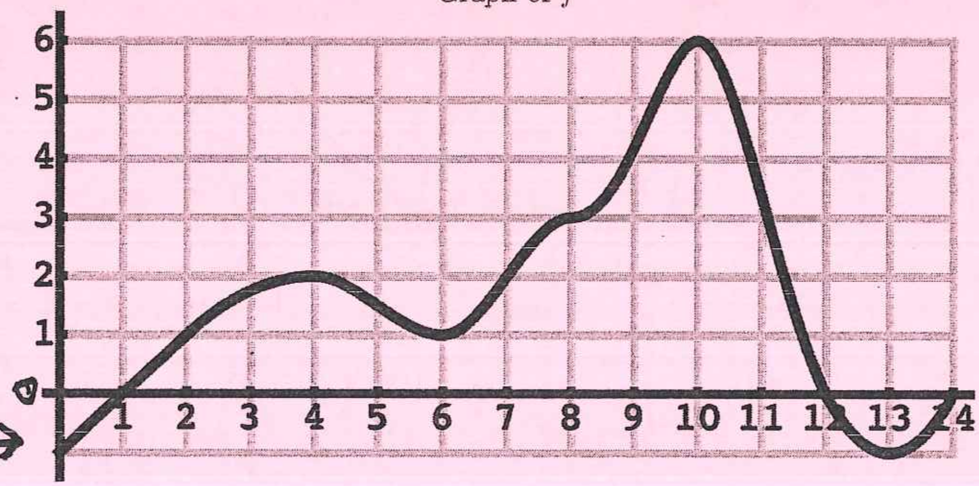
(b) $\frac{d}{dx}\left[\int_x^a f(t)\, dt\right] =$ __________

(c) $\frac{d}{dx}\left[\int_a^b f(t)\, dt\right] =$ __________

See 2016

13. (12 points) The function f is continuous on the closed interval $[0, 14]$. Here is its graph:

Graph of f



Note that the part of the graph of f over the interval $[0, 2]$ is a **line segment of slope 1 from the point** $(0, -1)$ **to the point** $(2, 1)$. Let N be the function defined by the formula

$$N(x) = \int_0^x f(t)\,dt \text{ for } 0 \le x \le 14.$$

Parts (a)–(i) are worth one point each. Parts (a)–(c) are short answer questions: Write the number requested in the box provided. Parts (d)–(i) are true-false questions: Circle T for true or F for false. Make sure that your choice is clear. You will not receive any credit for ambiguous answers.

(a) $N(0) =$ ________

(b) $N(1) =$ ________

(c) $N'(10) =$ ________

(d) T or F: The function N has a critical point at $x = 8$.

(e) T or F: The function N has an inflection point at $x = 10$.

(f) T or F: The function N is increasing on the interval $(0, 1)$.

(g) T or F: The function N has a local maximum at $x = 12$.

(h) T or F: The function N is decreasing on the interval $(12, 14)$.

(i) T or F: The function N has an inflection point at $x = 8$.

(j) (3 points) The final part of this problem **requires a one-sentence justification**. You will not receive any credit if you do not provide a valid justification or if your justification is longer than a typical one-sentence mathematical justification:

On what intervals is N concave up?

14. (1/4 point) Print your BU ID number on this page too: U______________

15. (4 points) Multiple choice: **Pick at most one answer.** You indicate your choice by filling in the circle to the immediate left of your choice. You will receive 4 points for the correct answer, 1 point if you do not pick any of the choices, and 0 points for the wrong answer. You will not receive any credit for an ambiguous answer.

Suppose that $f_1(x)$ and $f_2(x)$ are two functions that tend to infinity as $x \to \infty$. Recall that the notation $f_1 \ll f_2$ means that f_2 grows faster than f_1 as $x \to \infty$. Consider the four functions:

$$f_1(x) = e^x \qquad f_2(x) = \ln(3x) \qquad f_3(x) = x^{10} \qquad f_4(x) = 5^x$$

Which one of the following statements is true?

- ○ $f_2 \ll f_3 \ll f_4 \ll f_1$
- ○ $f_1 \ll f_2 \ll f_3 \ll f_4$
- ○ $f_2 \ll f_3 \ll f_1 \ll f_4$
- ○ $f_3 \ll f_4 \ll f_1 \ll f_2$
- ○ $f_4 \ll f_1 \ll f_2 \ll f_3$
- ○ $f_3 \ll f_2 \ll f_1 \ll f_4$
- ○ $f_4 \ll f_1 \ll f_3 \ll f_2$
- ○ $f_3 \ll f_2 \ll f_4 \ll f_1$
- ○ $f_2 \ll f_1 \ll f_3 \ll f_4$
- ○ $f_1 \ll f_2 \ll f_4 \ll f_3$

End of Exam

You may use the bottom of this page as well as the next page if you did not have enough space to answer a question, but **do not separate this page from the rest of the exam booklet.** If you want the graders to consider something that is written below this line or on the next page, make sure that there is a note to that effect next to the relevant problem. If you write something here that you would like the graders to ignore, cross it out.