SOLUTIONS FIRST PRACTICE TEST
(FALL 2021)

2) a) $\frac{dy}{dx} = -4e^{-4x}\cos(2x) - 2e^{-4x}\sin(2x)$

b) $\frac{dy}{dx} = \frac{\left(\frac{1}{x}\right)(x^2+5) - 2x\ln(3x)}{(x^2+5)^2}$

c) $\frac{dy}{dx} = 5e^{5x} + 3ex^{3e-1} + 0$

d) $\frac{dy}{dx} = \left(\frac{1}{1+\left(\sqrt{3+\sin x}\right)^2}\right)\cdot\left(\frac{\cos x}{2\sqrt{3+\sin x}}\right)$

4) u-sub: (not on our final, as stated earlier)

$$\int x^3 \sin x^4\, dx \underset{du=4x^3dx}{\overset{u=x^4}{=}} \frac{1}{4}\int \sin(u)\, du$$

$$= -\frac{1}{4}\cos u + C$$

(useful for MA124=Calc 2)

$$= \boxed{-\frac{1}{4}\cos(x^4) + C}$$

✓ derivative of ans. ⇒ $x^3\sin x^4$ ✓

SOLUTIONS

5) a) $\int_1^{e^5} \frac{1}{x}\,dx = \left(\ln|x| + C\right)\Big|_1^{e^5}$

FTC
$\int_a^b f(x)\,dx = F(b) - F(a)$

$= \ln|e^5| - \cancel{\ln|1|} \to 0$

$= \boxed{5}$

b) $\int_0^{1/2} \frac{3}{\sqrt{1-3x^2}}\,dx \underset{\frac{du}{\sqrt{3}} = dx}{\overset{u=\sqrt{3}x}{=}} \int_{u=0}^{u=\frac{\sqrt{3}}{2}} \frac{\sqrt{3}\,du}{\sqrt{1-u^2}}$

$= \sqrt{3}\,\sin^{-1} u \Big|_{u=0}^{u=\frac{\sqrt{3}}{2}}$

$= \boxed{\frac{\pi}{\sqrt{3}}}$ $\left(\sin^{-1}\frac{\sqrt{3}}{2} = \frac{\pi}{3}\right)$

6) a) $\boxed{8}$

b) $\boxed{7}$

c) $\boxed{3}$

the only correct answer

8) $\boxed{f''(1) < f(1) < f'(1)}$ because $f''(1) < 0$ conc. down, $f(1) = 0$, & $f'(1) > 0$ (slope of tan. line > 0)

(none of the others can be correct)

SOLUTIONS

9) $f(x) = \begin{cases} 4x-7 & \text{if } x<2 \\ 3 & \text{if } x=2 \\ 3-x & \text{if } x>2 \end{cases}$

$\boxed{T}$ the limit of $f(x)$ as $x \to 2^-$ exists ✓

$$\lim_{x\to 2^-} f(x) = \lim_{x \to 2^-} 4x-7 = 1. \checkmark$$

$\boxed{T}$ $\lim_{x\to 2} f(x)$ exists, because also

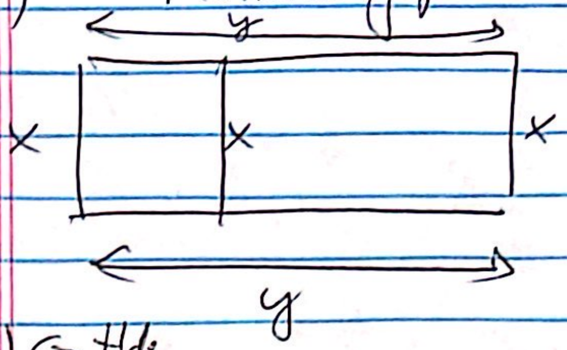
$$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 3-x = 1 \checkmark$$

The other two statements are false,
since $f(2) = 3 \neq \lim_{x\to 2} f(x)$
(3rd item on continuity check list is violated)

& also, if a function is NOT continuous @ a pt.
it cannot be differentiable there either.

SOLUTIONS

10) max. area of pen



(a) x meters = width of pen
y meters = length of pen

(a) cont'd:

Constraint: $3x+2y=60$ meters $\Rightarrow \boxed{y=\frac{-3}{2}x+30}$

(b) objective function: area $= xy$

$$\Rightarrow A(x)=x\cdot\left(-\tfrac{3}{2}x+30\right)$$

$$\Rightarrow \boxed{A(x)=\frac{-3}{2}x^2+30x}$$

domain $0<x<20$

$x=0$ ⇒ zero area (min.)

b/c need 3 pieces of length x

(c) set $A'(x)=0$

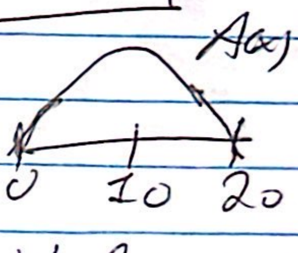
$$\Rightarrow -3x+30=0 \Rightarrow \boxed{x=10\text{ m}}$$

$$\Rightarrow \boxed{y=15\text{ m}}$$

(d) $A''(x)=-3 \Rightarrow A(x)$ is conc. down

$\Rightarrow x=10$ is a local max

(by 2nd deriv. test)



alt: use first deriv. test: $A'(x)>0$ just to left of $x=10$
& $A'(x)>0$ just to right of $x=10$

SOLUTIONS

12)

$$\frac{d}{dx}\ln(4x) = \frac{d}{dx}\ln(x)$$

$\boxed{\text{TRUE}}$ several ways to see this:

① $\ln(4x) = \ln(4) + \ln(x)$ (since $\ln(ab) = \ln(a)\times\ln(b)$ for $a,b>0$)

& so $\frac{d}{dx}\ln(4x) = \frac{d}{dx}(\ln 4) + \frac{d}{dx}(\ln x)$

$= 0 + \frac{d}{dx}(\ln x)$ ✓

② chain rule

$$\frac{d}{dx}\ln(4x) = \underbrace{\frac{1}{4x}}_{\text{deriv. of outer}} \cdot \underbrace{4}_{\text{deriv of inner}} = \frac{1}{x} = \frac{d}{dx}\ln(x)$$

③ observe $\ln(4x) - \ln(x) = \ln\left(\frac{4x}{x}\right)$

$\Rightarrow \ln(4x) - \ln(x) = \ln(4)$

& hence $\frac{d}{dx}\left(\ln(4x) - \ln(x)\right) = \frac{d}{dx}(\underbrace{\ln(4)}_{\text{const.}}) = 0.$

13) a) $\lim\limits_{x\to 2^+} \frac{x^2-9}{x^2+x-6} = \frac{-5}{0^+} = \boxed{-\infty}$

Cannot use l'Hôpital's rule

SOLUTIONS

13) b) $\lim_{x \to 0^+} (e^{4x} + x)^{1/x}$ $\quad 1^{\infty}$ form since $e^0 = 1$

First, find $\lim_{x \to 0^+} \ln(e^{4x}+x)^{1/x}$ & then exponentiate that result to get the final answer.

$$\lim_{x \to 0^+} \ln(e^{4x}+x)^{1/x} = \lim_{x \to 0^+} \frac{\ln(e^{4x}+x)}{x}$$

(using prop$^{\underline{y}}$ of $\ln(a^b) = b\ln(a)$ $(a>0)$)

$$\overset{\text{LHOP}}{=} \lim_{x \to 0^+} \frac{\left(\frac{4e^{4x}+1}{e^{4x}+x}\right)}{1}$$

$$= 5$$

So, final answer $= \boxed{e^5}$ ✓

13) c) $\lim_{x \to 1} \frac{\cos(\frac{\pi}{2}x)}{1-\sqrt{x}}$ $\quad \frac{0}{0}$ form

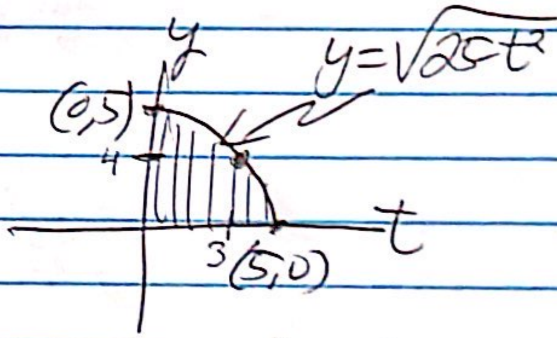
$$\overset{\text{LHOP}}{=} \lim_{x \to 1} \frac{-\frac{\pi}{2}\sin(\frac{\pi}{2}x)}{-\frac{1}{2\sqrt{x}}} = \frac{(-\frac{\pi}{2})(1)}{(-\frac{1}{2})} = \boxed{\pi}$$

SOLUTIONS (FIRST PRACTICE TEST)

15) $N(x)=\int_0^x \sqrt{25-t^2}\,dt$

$-5 \le x \le 5$

$\boxed{N(0)=0}$



$N(5)=\int_0^5 \sqrt{25-t^2}\,dt$

geo. $\Rightarrow N(5)=\frac{1}{4}\pi(5^2)$

$\Rightarrow \boxed{N(5)=\frac{25}{4}\pi}$

quarter-of circle, because

$y=\sqrt{25-t^2}$

$\Rightarrow y^2+t^2=5^2$ circle radius 5 about origin.

to find $N'(3)$, use FTC

$$\frac{d}{dx}\int_a^x f(t)\,dt = f(x)$$

$\Rightarrow N'(x)=\sqrt{25-x^2}$

so, $N'(3)=\sqrt{25-3^2}=\boxed{4}$