PRACTICE TEST for Midterm Examination 1
MA 123 – Fall 2024
This is an actual exam from a previous Fall semester

1. (3 points)

First Name : _____
Last (Family) Name : _____
BU ID number : _____

Directions

All of your work must be shown in this exam booklet.

Phones (of any type) must be turned OFF and in your backpack (silent mode is not permitted).

Backpacks and bags of any type must be stored in the front of the room. Keep your BU ID with you.

Books, notes, extra papers are not permitted.

The use of any electronic, mechanical, photonic, or quantum device to carry out any calculations or any steps in any part of the solutions is not permitted.

Please do not separate the pages of this exam booklet.

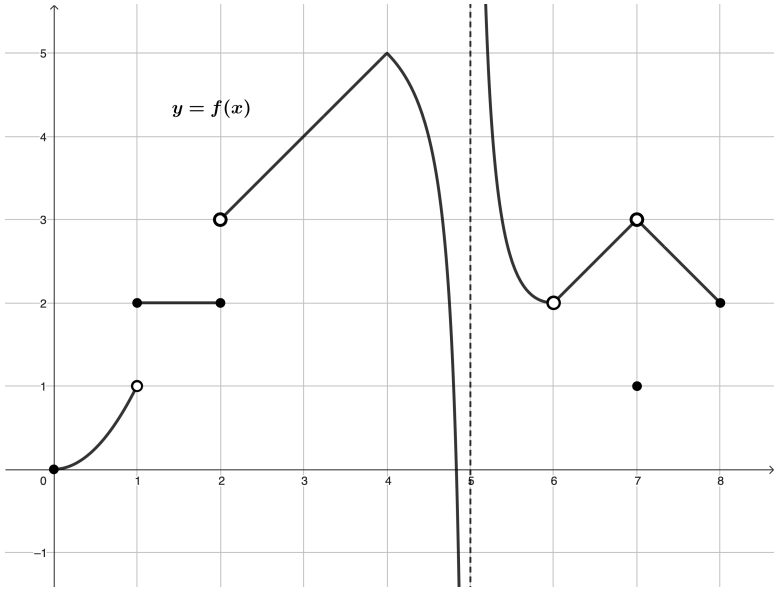
If you have a question about a problem, please raise your hand and a proctor will come to your seat to answer it.

Answers that are written logically and clearly will receive higher scores.

There are nine calculus problems on the exam and one logistical problem that simply asks for your first name, last (family) name, and BU ID number. The logistical problem is problem 1, at the top of this page. The calculus problems are numbered 2–10, starting on the second page of this exam booklet.

The entire exam booklet consists of 10 (ten) pages, double-sided, including this cover page. Please make sure that your exam booklet includes all 5 pieces of paper.

2. (7 points: 1 for each part) Here is the graph $y = f(x)$ of a function $f(x)$. Consider all $x \in [0, 8]$.



For each of the following limits, place your answer in the box. If the limit is a number, put the number in the box. If the limit is $+\infty$ or $-\infty$, write that in the box. If the limit does not exist and is not $\pm\infty$, then write DNE in the box.

a. $\lim_{x \to 1^-} f(x) =$ ☐

b. $\lim_{x \to 1^+} f(x) =$ ☐

c. $\lim_{x \to 6} f(x) =$ ☐

d. $\lim_{x \to 7} f(x) =$ ☐

e. $\lim_{x \to 3} f(x) =$ ☐

f. $\lim_{x \to 5^+} f(x) =$ ☐

g. $\lim_{x \to 2} f(x) =$ ☐

3. (12 points: 3 for each part) In this problem, you are to consider the function

$$s(t) = -16t^2 + 96t,$$

which gives the height (measured in feet) of a rock, thrown vertically straight up from the ground at time $t = 0$. Time ($t \geq 0$) is measured in seconds. For each of the following four parts, you must include **UNITS** in your answer and show enough work to justify your answer. You may use any method that is appropriate and that has been learned in class so far from Chapters 2.1–2.6 and 3.1–3.7.

a. Find the average velocity of the rock on the time interval $[1, 2]$.

b. Find the instantaneous velocity of the rock at time $t = 2$.

c. Find the time at which the velocity of the rock is equal to zero.

d. Find the slope of the tangent line to the graph of $y = s(t)$ at $t = 5$.

4. (16 points: 4 for each part)
For each of the following four limits, if it exists, evaluate it using only techniques learned so far in class this semester. If the limit is infinite, indicate if it is $+\infty$ or $-\infty$. If the limit does not exist and is NOT infinite, write DNE. Show enough work to justify your answer.

a. $\lim_{x \to 2} \dfrac{x^2 + x - 6}{x^2 + 2x - 8}$

b. $\lim_{x \to \frac{3}{4}^+} \dfrac{-x}{\sqrt{4x - 3}}$

c. $\lim_{x \to +\infty} \dfrac{7x^3 - x^2 + 4}{5x^3 + 5x + 1}$

d. $\lim_{x \to \frac{\pi}{2}^-} \tan(x)$

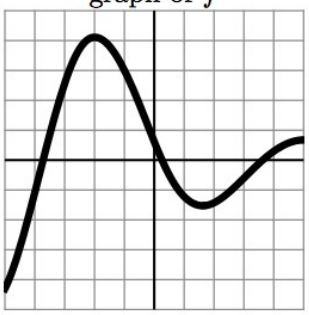
5. (8 points: 4 for each part)

a. Write down all of the INTERVALS on which the following function $f(x)$ is continuous:
$f(x) = \sqrt{36 - x^2}$

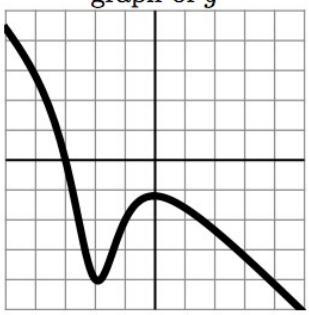
b. Write down all of the POINTS at which the following function $g(x)$ has discontinuities:
$g(x) = \dfrac{3x - 1}{3x^2 - 13x + 4}$

6. (9 points: 3 for each part) For each function that is graphed in parts (a)-(c), write in the box below it which graph (1-9) is the graph of its derivative.

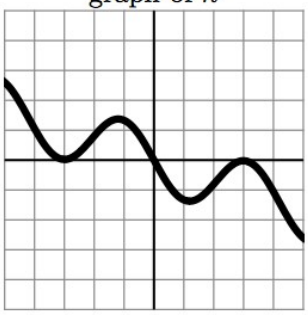
(a.) graph of f



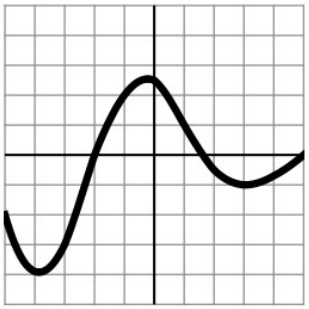
(b.) graph of g



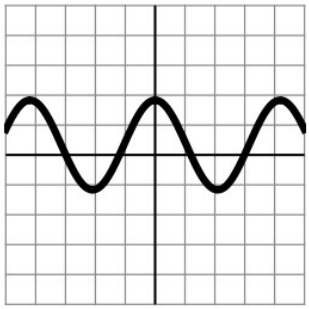
(c.) graph of h



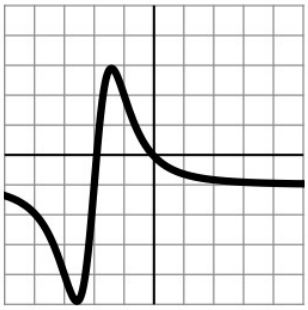
(1.)



(2.)



(3.)



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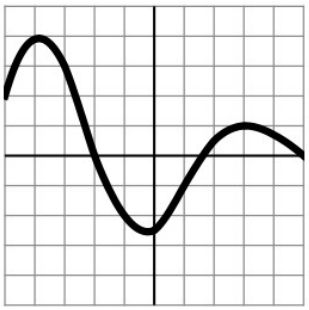

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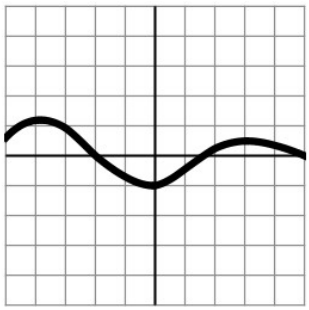

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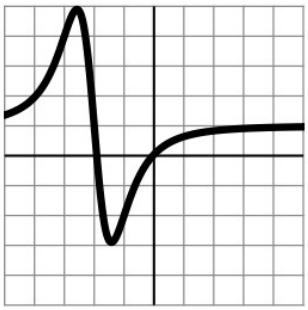
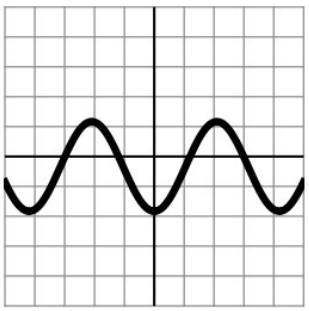
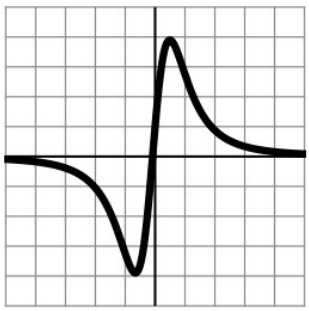


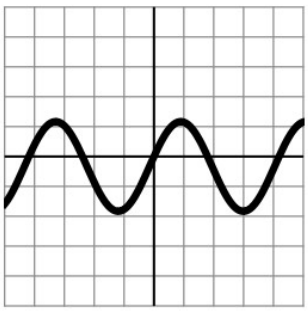
(7.)



(8.)



(9.)



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7. (12 points) In this problem, you are to consider the function

$$f(x) = \frac{x}{x+1}$$

Using only the definition of the derivative and the limit theorems learned so far this semester in the class, calculate $f'(2)$. You will not receive any credit if you do not start with the limit definition of $f'(x)$ or if you use limit theorems not studied so far this semester in class.

8. (12 points: 3 for each part) In each part, calculate the derivative $\frac{dy}{dx}$ by any appropriate method that we have learned so far. Please do NOT simplify your answers.

a. $y = \dfrac{e^{-x^2}}{x^3 + 5}$

b. $y = \pi^5$

c. $y = \tan\left(e^{\frac{1}{x}}\right)$

d. $y = \cos^3\left(x^2 \sin(x)\right)$

9. (9 points)

In this problem, you are working with the function $f(x) = \dfrac{2x^2}{x+1}$. Calculate an equation of the tangent line to the graph of $y = f(x)$ at $x = 1$. Write your answer either in slope-intercept form or in point-slope form, and show all of your work.

10. (12 points: 3 for each part) In this problem, you are given the following table of data about two functions $f(x)$ and $g(x)$ and their derivatives $f'(x)$ and $g'(x)$ at the values of x indicated across the top row of the table. In each part, you must state the value of the indicated derivative.

	$x=1$	$x=2$	$x=3$	$x=4$	$x=5$
$f(x)$	0	3	5	1	0
$f'(x)$	5	2	-5	-8	-10
$g(x)$	4	5	1	3	2
$g'(x)$	2	10	20	15	20

a. $(fg)'(2)$

b. $\left(\dfrac{f}{g}\right)'(3)$

c. $(f(g(x)))'(2)$

d. $(g(f(x)))'(4)$