MA 123 – PRACTICE TEST for Fall 2024
Practice test for Midterm 2

1. (3 points)

First Name : ______________________________

Last (Family) Name : ______________________________

BU ID number : ______________________________

Directions: The directions will be the same as on Midterm 1.

2. (15 points: 5 for each part) Without simplifying your answer in any part, write down the derivative of the given function in each part.

a. $f(x) = \pi^5 + \sin^{-1}\left(e^{-3x}\right)$

b. $g(x) = \tan^{-1}\left(e^{\cos x}\right)$

c. $h(x) = \cos^{-1}(x)$

3. (8 points) Sand falls from an overhead bin and accumulates into a pile that is in the shape of a cone with a radius that is always three times its height. You are told that the height of the pile increases at a rate of 2 inches per second. At what rate is the volume of the cone changing at the instant when the pile is 10 inches high? (Recall that the volume of a cone is $V = \frac{\pi}{3}r^2h$.).

Note for Fall 2024 actual midterm 2: For related rate problems in two dimensions, you need to know the formulas for areas of circles and rectangles, as well as the formula for distance between points.

4. (10 points: 1 for each value of x) You are to consider the function $f(x) = x^4 - 4x^3 + 4x^2$ on the interval $[-1, 3]$. Do not put more than one value of x in any box. (Some boxes in parts b–e may be empty.)

a. There are three critical points of $f(x)$. List the x value of each critical point (no more than one value per box): ☐ ☐ ☐

b. List the x value(s) of each local minimum (no more than one value per box and some boxes may be empty): ☐ ☐ ☐

c. List the x value(s) of each local maximum (no more than one value per box and some boxes may be empty): ☐ ☐ ☐

d. List the x value(s) of the point(s) at which $f(x)$ has its absolute minimum (no more than one value per box and one box may be empty): ☐ ☐

e. List the x value(s) of the point(s) at which $f(x)$ has its absolute maximum: (no more than one value per box and one box may be empty): [ ] [ ]

5. (9 points: 3 for each part) In this problem, you are given the function $f(x) = x - 2\tan^{-1}(x)$. You are also given the derivative: $f'(x) = 1 - \dfrac{2}{1+x^2}$.

a. Find the value of x for each of the critical points of $f(x)$.

b. Find the intervals on which $f(x)$ is increasing.

c. Find the intervals on which $f(x)$ is decreasing.

6. (12 points: 3 for each part) In this problem, you are given the function $f(x) = 3x^5 - 10x^4$ You are also given its derivative: $f'(x) = 15x^4 - 40x^3$

a. Calculate the second derivative $f''(x)$.

b. Find the interval(s) on which $f(x)$ is concave up.

c. Find the interval(s) on which $f(x)$ is concave down.

d. Find the point(s) of inflection of $f(x)$.

7. (10 points) A manufacturer produces a shipping crate with a square base and a volume of 54 m^3 (meters cubed). The material used for the top and bottom of the crate costs twice as much (per square meter) as the material used for the sides of the crate. Find the dimensions –including units– of the crate that minimize the cost to produce it.

8. (8 points total) In this problem, consider the function $f(x) = x^2 e^{-x}$. Use the number e in your answers.

a. (4 points) Write down the equation of the linear approximation $L(x)$ of f using $a = 1$,

b. (2 points) Find the linear approximation of $f(e/2)$.

c. (2 points) Being sure to state your reasoning, state whether the linear approximation in part b is an over estimate of $f(e/2)$ or an under estimate of $f(e/2)$. (Hint: $2 - 4e + e^2 < 0$.)

9. (15 points: 5 for each part)

In this problem, you are to evaluate the following three limits, showing your work for each.

a. $\displaystyle\lim_{x\to 1} \frac{\ln(x)}{17x - x^2 - 16}$

b. $\displaystyle\lim_{x\to 2} \frac{x^2 + x - 6}{\sqrt{8 - x^2} - 1}$

c. $\displaystyle\lim_{x\to 0^+} (1 + 4x)^{3/x}$

10. (10 points: 5 for each part)

a. Find an anti-derivative $F(x)$ of the following function: $f(x) = \sec^2(x)$

b. Find an anti-derivative $G(x)$ of the following function: $g(x) = \dfrac{1}{\sqrt{x}}$