Answer the questions in the spaces provided on the question sheets. **You must show your work to get credit for your answers. There will be eight problems on the actual final exam.**
1. (15 points) Use the method of elimination to find the complete solution of $Ax = b$, being sure to state the particular solution and the specific vector(s) in the null space, where $A = \begin{bmatrix} 1 & 2 & 3 & 5 \\ 2 & 4 & 8 & 12 \\ 3 & 6 & 7 & 13 \end{bmatrix}$ and $b = \begin{bmatrix} 0 \\ 6 \\ -6 \end{bmatrix}$. 
2. (a) (12 points) Use the method of elimination to compute $A^{-1}$ where 
\[
A = \begin{bmatrix}
2 & 1 & 1 \\
1 & 2 & 1 \\
1 & 1 & 2
\end{bmatrix}.
\]
(b) (3 points) Calculate the determinant of the following matrix $B$ and state why $B$ is not invertible.
\[
B = \begin{bmatrix}
3 & 0 & -3 \\
-2 & 0 & 4 \\
4 & 0 & 7
\end{bmatrix}.
\]
3. (15 points) In this problem, you are to consider the matrix $A = \begin{bmatrix} 3 & 4 & 2 & 6 \\ -3 & -6 & -4 & -2 \\ 3 & 8 & 9 & 1 \end{bmatrix}$. Find the rank $r(A)$, the column space $\text{Col}(A)$, and the null space $\text{Nul}(A)$.
4. (10 points) Let \( \mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}, \mathbf{v}_4 = \begin{bmatrix} 5 \\ 2 \\ 2 \end{bmatrix} \). Determine the maximum number of linearly independent vectors.
5. (15 points) In this problem, you are to consider the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$. Find the following spaces: $\text{Col}(A), \text{Nul}(A)$. 
6. (12 points)

\[ A = \begin{bmatrix} 1 & -6 \\ 2 & -6 \end{bmatrix}. \]

(a) (3 points) Find all of the eigenvalues of \( A \).

(b) (3 points) For each of the eigenvalues, find an associated eigenvector.

(c) (3 points) Sketch both eigenvectors.

(d) (3 points) Diagonalize \( A \), being sure to identify the eigenvector matrix \( P \) and the eigenvalue matrix \( D \).
7. (10 points) Determine whether or not \( b = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix} \) is a linear combination of the vectors \( v_1 = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix} \), \( v_2 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \), and \( v_3 = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix} \). Also, if \( b \) is a linear combination of these vectors, find that linear combination.
8. In this problem, you are to consider the matrices \( A_1 = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \) and \( A_2 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \).

(a) (2 points) Find and sketch the images of \( e_1 \) and \( e_2 \) under the mapping by the matrix \( A_1 \).

(b) (2 points) Describe in words how the matrix \( A_1 \) acts on general vectors \( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \); recall the words used in Tables 1-4 in Section 1.9.

(c) (2 points) Find and sketch the images of \( e_1 \) and \( e_2 \) under the mapping by the matrix \( A_2 \).

(d) (2 points) Describe in words how the matrix \( A_2 \) acts on general vectors \( \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \);

(e) (2 points) Write down the matrix that represents reflection across the line \( x_2 = -x_1 \).
9. Compute $A^{-1}$ where

(a) (3 points) $A = \begin{bmatrix} 3 & -9 \\ 2 & 6 \end{bmatrix}$, using any method you prefer.

(b) (7 points) $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, using row reduction.
10. Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}$, $v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}$, $v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}$, $v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$.

(a) (8 points) Find the maximum number of linearly independent vectors in $H = \text{span}\{v_1, v_2, v_3, v_4\}$.

(b) (2 points) Let $A$ be the matrix whose columns are given by the vectors $v_1, v_2, v_3, v_4$. Find $\text{rank}(A)$.
11. (10 points) Given \( A = \begin{bmatrix} 0 & 1 & -2 & 2 & 0 \\ -1 & 3 & 0 & 1 & 6 \\ -8 & -1 & 3 & 5 & 1 \end{bmatrix} \). Determine both \( \text{Col} (A) \) and \( \text{Nul} (A) \).
12. (10 points) Given $A = \begin{bmatrix} 6 & 1 & 0 & -1 \\ 2 & 2 & 0 & 1 \\ 0 & 3 & 8 & 0 \\ 0 & 1 & 0 & 5 \end{bmatrix}$. Calculate $\det(A)$. 


13. (10 points) Find all of the eigenvalues and their associated eigenvectors of the matrix

$$A = \begin{bmatrix} 4 & 1 \\ 3 & 6 \end{bmatrix}.$$
14. (10 points) Find all of the eigenvalues and their associated eigenvectors of the matrix

\[ A = \begin{bmatrix} 3 & 1 \\ -2 & 5 \end{bmatrix} . \]