Course on Geometric Singular Perturbation Theory  
Twente, NL, January 7–11, 2008  
Doelman-Kaper: some early homework problems

**Problem 1** — Find the three roots of the following equation by perturbation expansion up to and including terms of $O(\varepsilon^2)$:

$$\varepsilon x^3 - x + 1 = 0, \quad 0 < \varepsilon \ll 1.$$ 

**Problem 2** — When $\varepsilon = 0$, the following equation has the root $x = 1$:

$$(1 + \varepsilon)x = e^{x-1}.$$ 

Find the leading correction(s) to this root for $0 < \varepsilon \ll 1$.

**Problem 3** — For $0 < \varepsilon \ll 1$, find the inner and outer expansions of the solution of the following boundary value problem on the interval $x \in [0, 1]$:

$$\varepsilon y''(x) + \frac{1}{1+x} y'(x) - y(x) = 0,$$

subject to

$$y(0) = 0,$$
$$y(1) = e^{3/2}.$$ 

Can you also find the uniformly valid solution?

**Problem 4** — Consider the two-point boundary value problem on $x \in [0, 1]$ with $0 < \varepsilon \ll 1$:

$$\varepsilon y''(x) + y'(x) - x^\alpha y = 0, \quad y(0) = y(1) = 1.$$ 

It is known that the thickness of the boundary layer in this problem is $O(\varepsilon^3)$. Answer the following three questions: Where is the boundary layer? What is the value of $\alpha$? What is the outer solution?