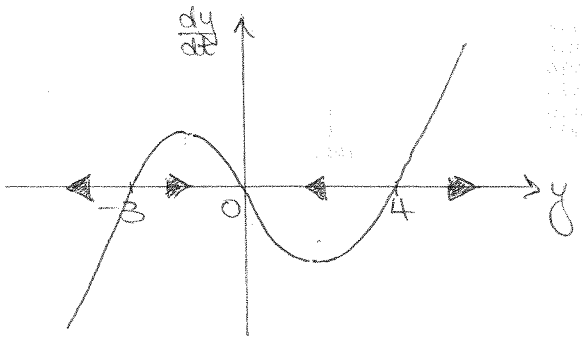
MA226 HW WEEK 02

SECTION 1.1, QUESTION 5

$$\frac{dy}{dt} = y(y^2 - y - 12) = y(y-4)(y+3).$$



a). y is in equilibrium for $y = -3, 0, 4$.

b). y is increasing for $-3 < y < 0$, $y > 4$.

c). y is decreasing for $y < -3$, $0 < y < 4$

SECTION 1.2, QUESTION 40

a). $$\frac{dC}{dt} = k_1(N - C) + k_2 E.$$

$$\Rightarrow \int \frac{dC}{k_1(N-C)+k_2 E} = \int dt$$

$$\Rightarrow \frac{1}{k_1}\int \frac{dC}{N-C+\frac{k_2}{k_1}E} = t + K_1, \quad K_1 \text{ arb. constant}.$$

$$\Rightarrow \frac{-1}{k_1}\ln\left(N-C+\frac{k_2}{k_1}E\right) = t + K_1.$$

$$\Rightarrow N - C + \frac{k_2}{k_1}E = e^{-k_1 t}\cdot A, \quad A := e^{k_1 K_1}.$$

Initial Condition: $C(0) = C_0$

$$\Rightarrow A = N + \frac{k_2}{k_1}E - C_0.$$

Thus, the solution is

$$C(t) = \left(N + \frac{k_2}{k_1}E\right)\left(1 - e^{-k_1 t}\right) + C_0 e^{-k_1 t}.$$

For $N = 200$, $k_1 = 0.1$, $k_2 = 0.1$, $E = 400$, and $C_0 = 150$, we have

$$C(t) = 600\left(1 - e^{-0.1t}\right) + 150 e^{-0.1t}.$$

The person's cholesterol level after 2 days is $C(2) = 600\left(1 - e^{-0.2}\right) + 150 e^{-0.2} \approx 231.571$.

NOTE: The ODE is linear, so could be solved using integrating factors.

c). $$\lim_{t\to\infty} C(t) = \lim_{t\to\infty}\left(\left(N + \frac{k_2}{k_1}E\right)\left(1 - e^{-k_1 t}\right) + C_0 e^{-k_1 t}\right) = N + \frac{k_2}{k_1}E = 600.$$

Section 1.8, Question 33

b). $$\frac{dy}{dt} = a(t)\,y + b(t) \qquad \text{— (i).}$$

y_p is a solution of (i) $\Rightarrow \dfrac{dy_p}{dt} = a(t)\,y_p + b(t) \qquad$ — (ii)

y_q is a solution of (i) $\Rightarrow \dfrac{dy_q}{dt} = a(t)\,y_q + b(t) \qquad$ — (iii)

Let $y = y_p - y_q$. Then

$$\frac{dy}{dt} = \frac{d}{dt}\left(y_p - y_q\right) = \frac{dy_p}{dt} - \frac{dy_q}{dt}$$

$$= a(t)\,y_p + b(t) - \left(a(t)\,y_q + b(t)\right), \text{ by (ii) and (iii).}$$

$$= a(t)\left(y_p - y_q\right)$$

$$= a(t)\,y$$

Thus, $y = y_p - y_q$ is a solution of the homogeneous problem.

—//—

Section 1.9, Question 16

$$\frac{dy}{dt} - y = 4\cos t^2.$$

Integrating Factor, $I(t) = e^{\int(-1)dt} = e^{-t}$.

Multiplying the ODE by $I(t)$:

$$e^{-t}\frac{dy}{dt} - e^{-t}y = 4e^{-t}\cos(t^2)$$

$$\Rightarrow \frac{d}{dt}\left(e^{-t}y\right) = 4e^{-t}\cos t^2.$$

$$\Rightarrow e^{-t}y = 4\int e^{-t}\cos t^2\,dt + C., \quad C \text{ arb. constant.}$$

$$\Rightarrow y = 4e^{t}\int e^{-t}\cos t^2\,dt + Ce^{t}.$$

NOTE: $\int e^{-t}\cos t^2\,dt$ cannot be evaluated explicitly. It can be expressed in terms of the imaginary error function, erfi, which is a non-standard (but commonly occurring) function.

—//—