MA 226 HW WEEK 04

SECTION 1.6, QUESTION 40

$$\frac{d\theta}{dt} = 1-\cos\theta + I(1+\cos\theta) = 1-\cos\theta - \frac{1}{3}(1+\cos\theta), \quad I = -\frac{1}{3}.$$

a). Equilibria: $\frac{d\theta}{dt} = 0 \Rightarrow \frac{2}{3} - \frac{4}{3}\cos\theta = 0$

$$\Rightarrow \cos\theta = \frac{1}{2}.$$

$$\Rightarrow \theta = \pm\frac{\pi}{3} + 2k\pi, \ k \in \mathbb{Z}.$$

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b). Stability: Let $f(\theta) = 1-\cos\theta - \frac{1}{3}(1+\cos\theta) = \frac{2}{3} - \frac{4}{3}\cos\theta$.

Then $\frac{\partial f}{\partial \theta} = \frac{4}{3}\sin\theta$. — 1 PT

At $\theta = -\frac{\pi}{3}+2k\pi$, $\frac{\partial f}{\partial \theta} = \frac{4}{3}\sin\left(-\frac{\pi}{3}+2k\pi\right) = -\frac{2\sqrt{3}}{3} < 0 \Rightarrow \theta = -\frac{\pi}{3}+2k\pi$ IS STABLE

At $\theta = \frac{\pi}{3}+2k\pi$, $\frac{\partial f}{\partial \theta} = \frac{4}{3}\sin\left(\frac{\pi}{3}+2k\pi\right) = \frac{2\sqrt{3}}{3} > 0 \Rightarrow \theta = \frac{\pi}{3}+2k\pi$ IS UNSTABLE.

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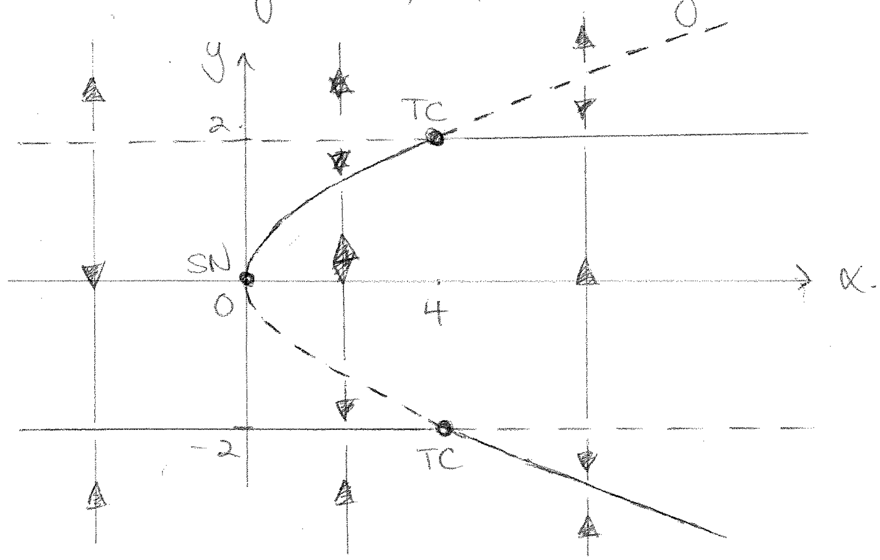
SECTION 1.7, QUESTION 5

$$\frac{dy}{dt} = (y^2-\alpha)(y^2-4)$$

Equilibria: $\frac{dy}{dt} = 0 \Rightarrow y = \pm\sqrt{\alpha}, \ \alpha \geq 0$ AND $y = \pm 2$.



Saddle-Node Bifurcation at $(y,\alpha) = (0,0)$

Transcritical Bifurcations at $(y,\alpha) = (\pm 2, 4)$.

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