MA 226 HW WEEK 05

HANDOUT (13/02/17), QUESTION 4

$$\frac{dv}{dt} = v - \frac{v^3}{3} - w. =: f(v,w)$$

(i). Equilibria: $f(v,w) = 0 \Rightarrow w = v - \frac{v^3}{3}$.

Stability: $\frac{\partial f}{\partial v} = 1 - v^2$

Bifurcation points occur when $f(v,w) = 0$ and $\frac{\partial f}{\partial v}(v,w) = 0$.

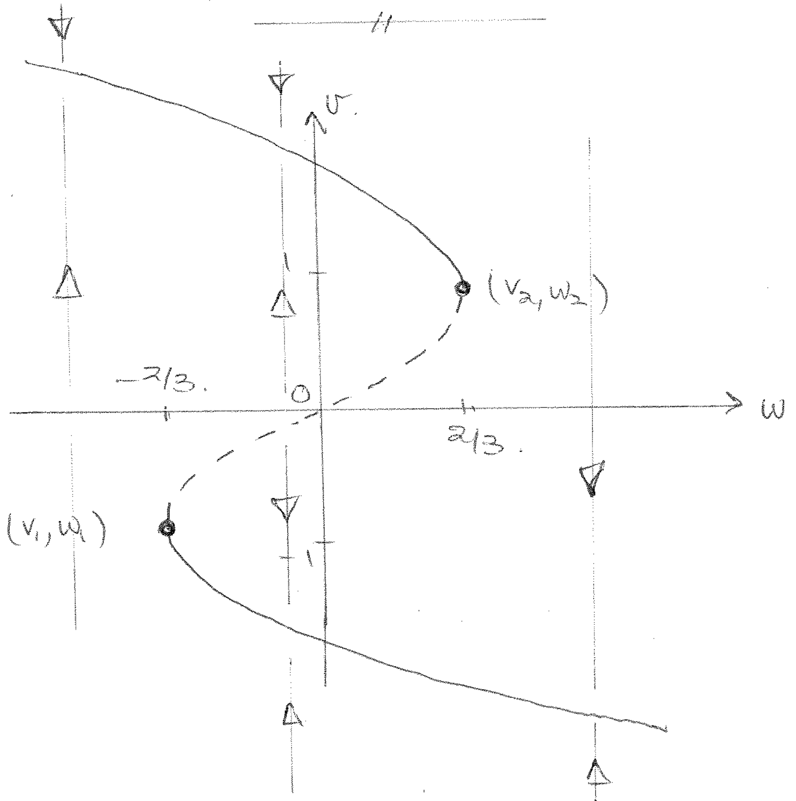
i.e., $\left.\begin{aligned} v - \frac{v^3}{3} - w &= 0 \\ 1 - v^2 &= 0 \end{aligned}\right\} (v_1, w_1) = (-1, -\tfrac{2}{3}),\ (v_2, w_2) = (1, \tfrac{2}{3})$

Classification: $\frac{\partial f}{\partial w} = -1 \neq 0 \quad \forall (v,w)$

$\frac{\partial^2 f}{\partial v^2} = -2v \neq 0$ for $v \neq 0$.

$\Rightarrow (v_1, w_1)$ and (v_2, w_2) are saddle-node bifurcations.

(ii).



SECTION 2-2, QUESTION 19

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} - 3x + x^3 = 0. \qquad — (*)$$

Let $v = \frac{dx}{dt}$. Then $\frac{dv}{dt} = \frac{d^2x}{dt^2}$.

Substituting into the ODE:

$$\frac{dv}{dt} + 2v - 3x + x^3 = 0.$$

Thus, the second-order equation $(*)$ is equivalent to the first-order 2D system:

$$\frac{dx}{dt} = v.$$

$$\frac{dv}{dt} = 3x - 2v - x^3.$$

(a). The vector field is $\underline{f} = \begin{pmatrix} v \\ 3x - 2v - x^3 \end{pmatrix}$.

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(b). Equilibria: $\dot{x} = 0 \Rightarrow v = 0$.

$\dot{v} = 0 \Rightarrow 0 = x(3 - x^2) \Rightarrow x = 0, \pm\sqrt{3}$.

Thus, the equilibria are $(x_0, v_0) = (0,0)$, $(x_1, v_1) = (-\sqrt{3}, 0)$, $(x_2, v_2) = (\sqrt{3}, 0)$.

NOTE: An eqm of $(*)$ is a point where there is no motion in x, i.e. $\frac{dx}{dt} = 0$, so it is not surprising that $v = 0$ for all equilibria.

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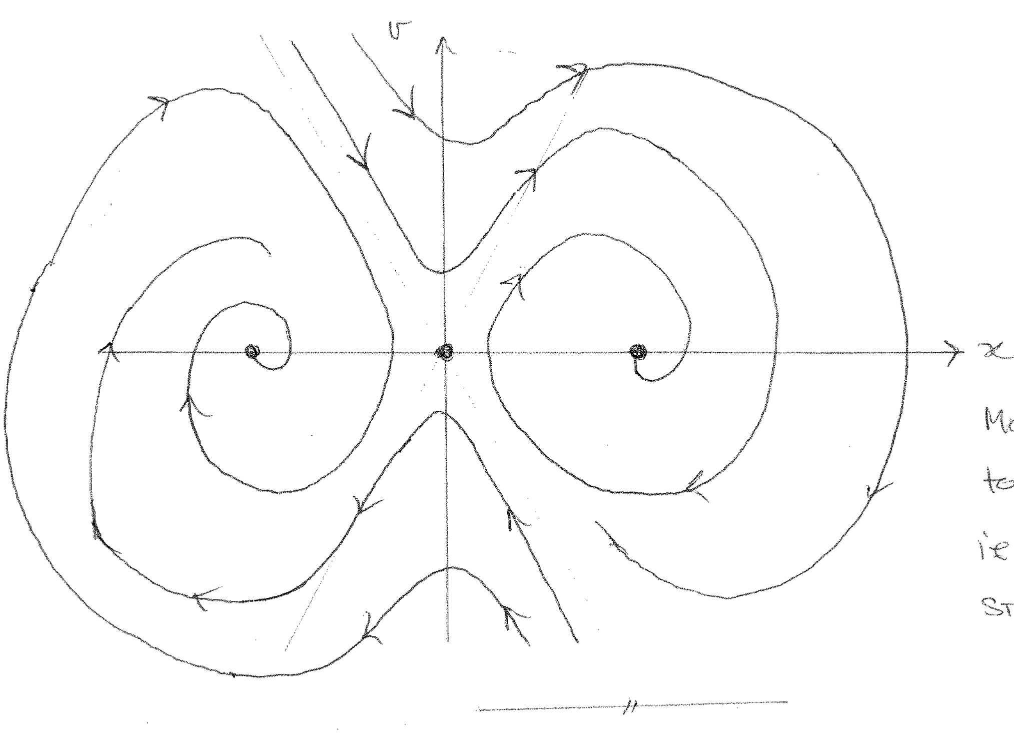
(c,d)



Most initial conditions converge to the equilibria at $(\pm\sqrt{3}, 0)$, i.e., (x_1, v_1) and (x_2, v_2) are STABLE. (x_0, v_0) is UNSTABLE.

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