MA 226 HW WEEK 07

SECTION 3.1, QUESTION 35

a). $y(t) := \begin{pmatrix} te^{2t} \\ -(t+1)e^{2t} \end{pmatrix}$

$$\Rightarrow \frac{dy}{dt} = \begin{pmatrix} \frac{d}{dt}(te^{2t}) \\ \frac{d}{dt}(-(t+1)e^{2t}) \end{pmatrix} = \begin{pmatrix} 2te^{2t} + e^{2t} \\ -2(t+1)e^{2t} - e^{2t} \end{pmatrix}$$

Now, $\begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} y = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} te^{2t} \\ -(t+1)e^{2t} \end{pmatrix}$

$$= \begin{pmatrix} te^{2t} + (t+1)e^{2t} \\ te^{2t} + 3(t+1)e^{2t} \end{pmatrix}$$

$$= \begin{pmatrix} 2te^{2t} + e^{2t} \\ -2(t+1)e^{2t} - e^{2t} \end{pmatrix}$$

Thus, $\frac{dy}{dt} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} y$, so y is a solution.

---

b). Let $y = \begin{pmatrix} te^{2t} \\ -(t+1)e^{2t} \end{pmatrix}$. We showed that y is a solution of $\dot{y} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} y$.

$\Rightarrow$ Cy is also a solution for any $C \in \mathbb{R}$.

NOTE: $Cy(0) = C\begin{pmatrix} 0 \\ -1 \end{pmatrix} \Rightarrow C = -2$ gives the desired initial condition.

Thus, the solution of $\dot{y} = \begin{pmatrix} 1 & -1 \\ 1 & 3 \end{pmatrix} y$, $y(0) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$ is

$$y = -2\begin{pmatrix} te^{2t} \\ -(t+1)e^{2t} \end{pmatrix}.$$

SECTION 3.2, QUESTION 17

$$B = \begin{pmatrix} a & b \\ b & d \end{pmatrix}$$

$$p(\lambda) = \det(B - \lambda I) = (a-\lambda)(d-\lambda) - b^2 = \lambda^2 - (a+d)\lambda + ad - b^2.$$

$$\Rightarrow \lambda_{1,2} = \frac{1}{2}\left(a+d \pm \sqrt{(a+d)^2 - 4(ad-b^2)}\right).$$

$$\Rightarrow \lambda_{1,2} = \frac{1}{2}\left(a+d \pm \sqrt{(a-d)^2 + 4b^2}\right).$$

Observe that $(a-d)^2 + 4b^2 \geq 0 \ \forall \ a, b, d$. Thus, the eigenvalues are always real.

Repeated eigenvalue occurs if $\lambda_1 = \lambda_2$, i.e.,

$$\frac{1}{2}\left(a+d + \sqrt{(a-d)^2+4b^2}\right) = \frac{1}{2}\left(a+d - \sqrt{(a-d)^2 + 4b^2}\right)$$

$$\Rightarrow (a-d)^2 + 4b^2 = 0.$$

$$\Rightarrow a = d \text{ and } b = 0.$$

Thus, if $b \neq 0$, we are guaranteed that $\lambda_1 \neq \lambda_2$.
