MA 294 - Turn in #4

(1) Let $f(x) = x^3 + 3x^2 - 2x - 4$ and $g(x) = x^2 + 2x + 1$ which are polynomials in $\mathbb{Z}[x]$.

It turns out that one can find polynomials $q(x), r(x) \in \mathbb{Z}[x]$ such that $f(x) = q(x)g(x) + r(x)$.

(a) Determine $q(x)$ and $r(x)$.

(b) By reducing coefficients mod p (for p a prime) one gets polynomials $\overline{f}(x), \overline{g}(x) \in \mathbb{Z}_p[x]$.

Is there a p such that $\overline{g}(x)$ evenly divides $\overline{f}(x)$?

[10 points]

(2) For a, b in a commutative ring

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$$

which is, of course, the Binomial Theorem.

If $a^p = 0$ and $b^q = 0$ for positive integers p, q find the least n which *guarantees* that $(a+b)^n = 0$.

[10 points]