Turn in #2

Given

$$R = \mathbb{Z}[\sqrt{2}] = \{a + b\sqrt{2} | a, b \in \mathbb{Z}\}$$

let $\phi : R \to R$ be a non-trivial homomorphism.
(i.e. one that does not simply send all elements to 0)

(a) Show that $\phi(1) = 1$ and therefore $\phi(a) = a$ for any $a \in \mathbb{Z}$.

(b) Use (a) to show that $\phi(a + b\sqrt{2}) = a + b\phi(\sqrt{2})$ for $a, b \in \mathbb{Z}$.

(c) If $\phi(\sqrt{2}) = c + d\sqrt{2}$, then determine the possible values of $c, d$.

[The answer to (a) actually plays a subtle role in the answer to this as does the fact that $c, d \in \mathbb{Z}$.]

**[10 points]**