

Turn in #7

(1) Let  $E$  be an extension field of  $F$  and let  $\alpha \in E$  be algebraic of odd degree over  $F$  (i.e.  $[F(\alpha) : F]$  is odd).

Show that  $\alpha^2$  is also algebraic over  $F$  and that  $F(\alpha) = F(\alpha^2)$ .

**[5 points]**

(2) Find the minimal polynomial  $p(x)$  for  $\alpha = \sqrt{3 - \sqrt{6}}$  over  $\mathbb{Q}$  and prove that it *is* irreducible.

Is  $\mathbb{Q}(\alpha)$  a splitting field for  $p(x)$ ? Explain.

**[15 points]**