$$\text{Why } 1+2+\cdots+n = \frac{n(n+1)}{2}$$

There are actually several ways of proving this:

INDUCTION

If $P(n)$ is the statement that $1+2+\cdots+n = \frac{n(n+1)}{2}$ then we first verify that $P(1)$ is true by noting that

$$1 = \frac{1(1+1)}{2}$$

and then (and here is the interesting part) suppose we assume that $P(n)$ is true, namely that

$$1+2+\cdots+n = \frac{n(n+1)}{2}$$

as such, let's add $n+1$ to both sides of this equation to yield

$$\begin{aligned} 1+2+\cdots+n+(n+1) &= \frac{n(n+1)}{2} + (n+1) \\ &\downarrow \\ &= \frac{n(n+1)}{2} + \frac{2(n+1)}{2} \\ &= \frac{n^2+3n+2}{2} \\ &= \frac{(n+1)(n+2)}{2} \end{aligned}$$

That is, we have shown $1+2+\cdots+(n+1) = \frac{(n+1)(n+1+1)}{2} = \frac{(n+1)(n+2)}{2}$ which is precisely the statement $P(n+1)$. Thus, since $P(1)$ is true and $P(n)$ implies $P(n+1)$ is true (*for any n!*) then the theorem is true for **all** n.

ALGEBRAIC PROOF

If $t_n = 1+2+\cdots+n$ then we can add t_n to itself to get

$$\begin{aligned} t_n + t_n &= 1+2+\cdots+n+1+2+\cdots+n \\ &= 1+2+\cdots+n+n+(n-1)+\cdots+1 \\ &= (1+n)+(2+n-1)+\cdots+(n+1) \\ &= (n+1)+(n+1)+\cdots+(n+1) \\ &= n(n+1) \end{aligned}$$

Thus $2t_n = n(n+1)$ and so $t_n = \frac{n(n+1)}{2}$ as claimed!

This was essentially the technique used by the mathematician Gauss when asked by his grade school teacher to add up the numbers from 1 to 100.
See *http://en.wikipedia.org/wiki/Carl_Friedrich_Gauss* for more details.