Ch. 4 #25

Let $X_d = \{\sigma \in D_n \mid |\sigma| = d\}$ Assume $n \geq 3$

We wish to show (for $d \neq 2$ and $d \mid n$) that $|X_d| = \phi(d)$

If $d = 1$ then $X_d = \{R_0\}$ and so $|X_d| = 1 = \phi(1)$ ✓

If $d > 2$ then X_d contains rotations only since the only elements of D_n of order > 2 are rotations.

The rotations of D_n all lie in the cyclic subgroup generated by $R_{\frac{360}{n}}$

Note that
$$|R_{\frac{360}{n}}| = |\langle R_{\frac{360}{n}} \rangle| = n$$

and since $\langle R_{\frac{360}{n}} \rangle$ is cyclic of order n, it contains $\phi(d)$ elements of order d (for $d \mid n$) by Theorem 4.4 in the text.)

Thus $|X_d| = \phi(d)$.

As for elements of order 2, D_n has n of them since each is a flip with respect to a vertex of an n-gon hence one for each of the n vertices.