Ch. 4 #40

$\langle m\rangle = \{mk \mid k\in\mathbb{Z}\} = \{\text{multiples of } m\}$
$\langle n\rangle = \{nk \mid k\in\mathbb{Z}\} = \{\text{multiples of } n\}$

So $\langle m\rangle \cap \langle n\rangle = \{\text{(common) multiples of } m \text{ and } n\}$

In particular

$$\operatorname{lcm}(m,n) \in \langle m\rangle \cap \langle n\rangle.$$

Claim: $\langle m\rangle \cap \langle n\rangle = \langle \operatorname{lcm}(m,n)\rangle$

Note
$\langle \operatorname{lcm}(m,n)\rangle = \{\text{multiples of } \operatorname{lcm}(m,n)\}$

$x \in \langle m\rangle \cap \langle n\rangle \to m \mid x$ and $n \mid x$
$\to \operatorname{lcm}(m,n) \mid x$
$\to x \in \langle \operatorname{lcm}(m,n)\rangle$

$x \in \langle \operatorname{lcm}(m,n)\rangle \to \operatorname{lcm}(m,n) \mid x$
$\to m \mid x$ and $n \mid x$
$\to x \in \langle m\rangle \cap \langle n\rangle$