Ch.2 #15. *If a,b are elements of an Abelian group G and n is an integer, then*

$$(ab)^n = a^n b^n$$

Proof. This is a straightforward proof by induction. We shall prove this for $n \geq 1$. (A similar argument works for $n \leq 1$.)

For $n = 1$ we have

$$(ab)^1 = a^1 b^1$$

i.e.

$$(ab) = ab$$

Now assume it's true for a given n, i.e.

$$(ab)^n = a^n b^n$$

then we would like to show that $(ab)^{n+1} = a^{n+1}b^{n+1}$. To conclude this, we proceed as follows:

$$\begin{aligned} (ab)^n &= a^n b^n \\ (ab)(ab)^n &= (ab)a^n b^n \\ (ab)^{n+1} &= aba^n b^n \\ &= aa^n bb^n (since\ G\ is\ Abelian\ ba^n = a^n b) \\ &= a^{n+1} b^{n+1} \end{aligned}$$

And we are done. □

[Note, this is also how you answer question 19, although for 19 you do **NOT** need to assume G is abelain.]