3.6 #13

$$f(x) + x^2[f(x)]^3 = 10 \qquad f(1) = 2$$

Find $f'(1)$

$$\downarrow$$

$$f'(x) + 2x[f(x)]^3 + x^2 3[f(x)]^2 \cdot f'(x) = 0$$

$x = 1$

$$f'(1) + 2(f(1))^3 + 1 \cdot 3(f(1))^2 f'(1) = 0$$

$$f'(1) + 2(2)^3 + 3(2)^2 f'(1) = 0$$

Solve for $f'(1)$ $\rightarrow$ $\boxed{f'(1) = \frac{-16}{13}}$

3.6 # 37

$$f(x) = e^x - x^2 \arctan x$$

$$\downarrow$$

$$f'(x) = e^x - \left(2x \arctan x + x^2 \frac{1}{1+x^2}\right)$$

3.6 # 51 Find all points on the curve $x^2y^2+xy=2$ when the tangent line has slope -1.

$$x^2y^2+xy=2$$

$$\downarrow$$

$$2xy^2+x^2\left(2y\frac{dy}{dx}\right)+y+x\frac{dy}{dx}=0$$

$$(2xy^2+y)+(2x^2y+x)\frac{dy}{dx}=0$$

$$\boxed{\frac{dy}{dx}=\frac{-2xy^2-y}{2x^2y+x}}$$

$$\frac{dy}{dx}=-1 \longrightarrow -1=\frac{-2xy^2-y}{2x^2y+x}$$

$$\downarrow$$

$$1=\frac{2xy^2+y}{2x^2y+x}$$

$$\downarrow$$

$$2x^2y+x=2xy^2+y$$

$$2xy^2+y-2x^2y-x=0$$

$$\boxed{2xy^2+(1-2x^2)y-x=0}$$

$$y=\frac{-(1-2x^2)\pm\sqrt{(1-2x^2)^2-4(2x)(-x)}}{4x}$$

$$=\frac{(2x^2-1)\pm\sqrt{(1-2x^2)^2+8x^2}}{4x}$$

$$(1-2x^2)^2+8x^2$$

$$=1-4x^2+4x^4+8x^2$$

$$=4x^4+4x^2+1$$

$$=(2x^2+1)^2$$

$$y=\frac{(2x^2-1)\pm\sqrt{(2x^2+1)^2}}{4x}$$

$$=\frac{(2x^2-1)\pm(2x^2+1)}{4x}$$

$$=\begin{cases}\dfrac{-2}{4x}=-\dfrac{1}{2x}\\[2ex]\dfrac{4x^2}{4x}=x\end{cases}$$

$$\Longrightarrow$$

$$x^2y^2 + xy = 2$$

$$y = -\frac{1}{2x}$$

$$x^2\left(-\frac{1}{2x}\right)^2 + x\left(-\frac{1}{2x}\right) = 2$$

$$x^2\frac{1}{4x^2} - \frac{1}{2} = 2$$

$$\frac{1}{4} - \frac{1}{2} = 2$$

Not possible

$$y = x$$

$$x^2x^2 + xx = 2$$

$$x^4 + x^2 = 2$$

$$x^4 + x^2 - 2 = 0$$

$$u = x^2$$

$$u^2 + u - 2 = 0$$

$$(u-1)(u+2) = 0$$

$$u = 1, -2$$

$$x^2 = 1 \rightarrow \boxed{x = \pm 1}$$

$$x^2 = -2 \text{ impossible}$$

Finally!

$$x = 1 \rightarrow y = 1$$

$$x = -1 \rightarrow y = -1$$

ie $(1,1)$ and $(-1,-1)$

DONE!