

$$\tan^2 x + 1 = \sec^2 x$$

$$\int \sec^3 x \, dx = \int \sec x \sec^2 x \, dx$$

$$u = \sec x \quad dv = \sec^2 x \, dx$$

$$du = \sec x \tan x \, dx \quad v = \tan x$$

$$= \sec x \tan x - \int \sec x \tan^2 x \, dx$$

$$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$$

$$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$$

$$2 \int \sec^3 x \, dx = \sec x \tan x + \int \sec x \, dx$$

$$\int \sec^3 x \, dx = \frac{1}{2} \sec x \tan x + \frac{1}{2} \int \sec x \, dx$$

$$= \frac{1}{2} \sec x \tan x + \frac{1}{2} \ln |\sec x + \tan x| + C$$