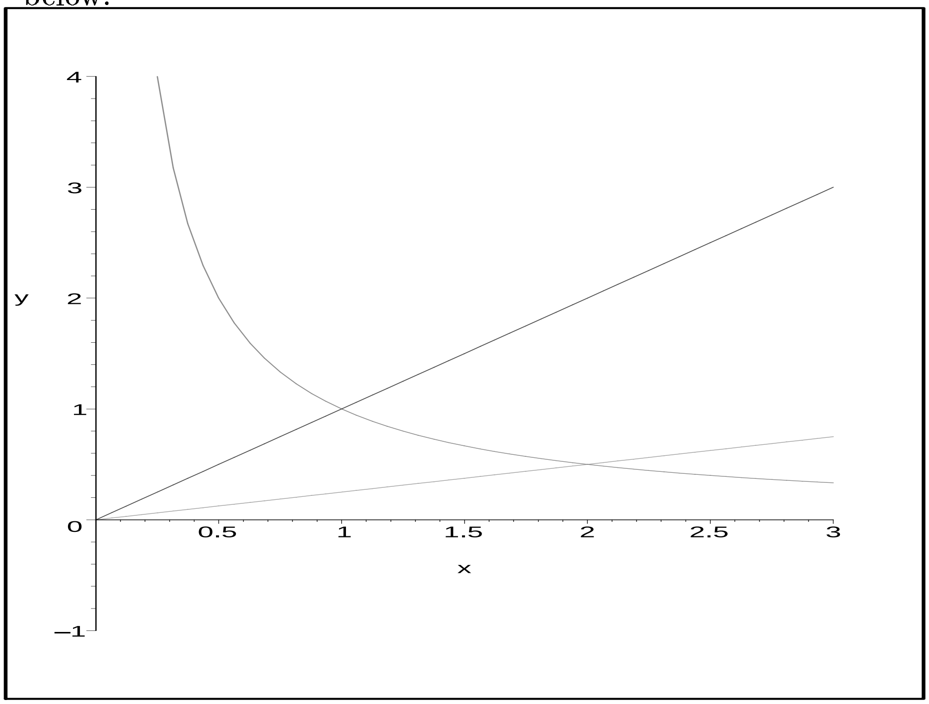
6.1 #15

The curves $y = 1/x$, $y = x$ and $y = \frac{1}{4}x$ form a triangular shaped region as shown below:



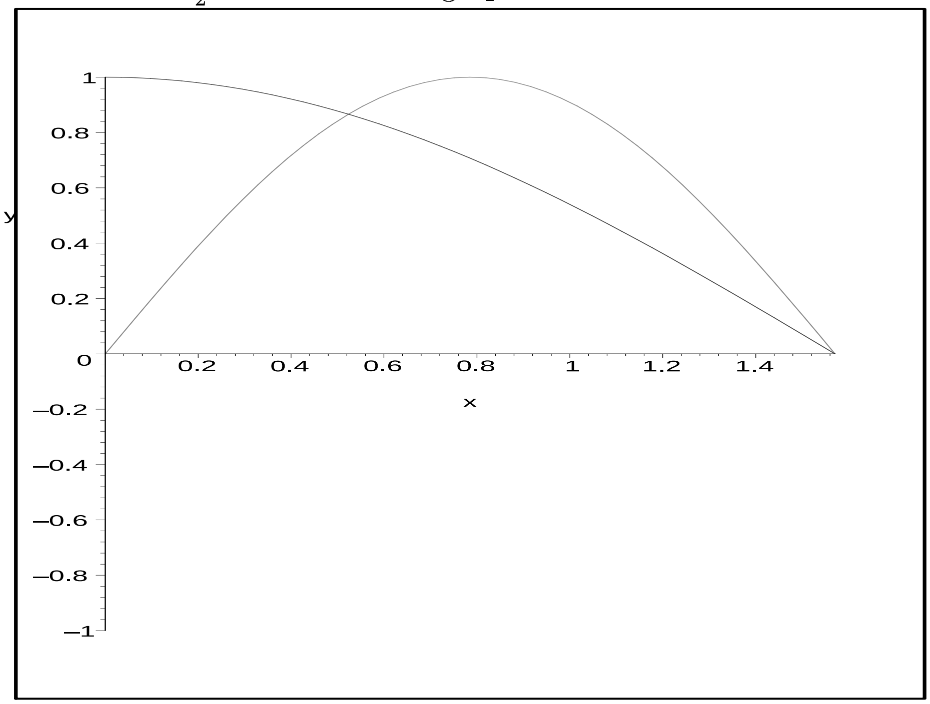
The points of intersection are $(0,0)$, $(1,1)$ and $(2,1/2)$ and so two integrals are necessary as the upper boundary of the region changes at $x = 1$ from $y = x$ to $y = 1/x$:

$$A = \int_0^1 \left(x - \frac{1}{4}x\right)\ dx + \int_1^2 \left(1/x - \frac{1}{4}x\right) dx$$

I'm not interested in the number, just the setup of the integrals.

6.1 #27

The graph of $y = sin(2x)$ looks like that of $y = sin(x)$ except that the behavior that normally takes place between 0 and $\frac{\pi}{2}$ instead takes place in $[0, \frac{\pi}{4}]$. (i.e. $sin(2x)$ has a maximum at $\frac{\pi}{4}$ instead of $\frac{\pi}{2}$. Recall that $cos(x)$ is 1 at $x = 0$ and decreases to 0 at $x = \frac{\pi}{2}$. These two are graphed below:



The points of intersection are when $x = \frac{\pi}{2}$ of course and $x = \frac{\pi}{6}$ (i.e. 30 degrees) since $sin(60^o) = cos(30^o)$ and inside $[0, \frac{pi}{3}]$ the graph of $cos(x)$ lies above the graph of $y = sin(2x)$ and the reverse is true inside $[\frac{\pi}{3}, \frac{\pi}{2}]$ so two integrals are required:

$$A = \int_0^{\frac{\pi}{3}} cos(x) - sin(2x)\ dx + \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} sin(2x) - cos(x)\ dx$$