MA 294 - Turn in #4

(1) Let $H = \begin{bmatrix} \vec{h}_1 & \dots & \vec{h}_m \end{bmatrix}$ be a binary matrix with columns $\vec{h}_i$.
Let C be the code that is the nullspace of H.
If

$$\begin{aligned}
&\{\vec{h}_1 + \vec{h}_2, \vec{h}_1 + \vec{h}_3, \dots, \vec{h}_1 + \vec{h}_m,\\
&\vec{h}_2 + \vec{h}_3, \vec{h}_2 + \vec{h}_4, \dots, \vec{h}_2 + \vec{h}_m,\\
&\vdots\\
&\vec{h}_{m-2} + \vec{h}_{m-1}, \vec{h}_{m-2} + \vec{h}_m\\
&\vec{h}_{m-1} + \vec{h}_m\}
\end{aligned}$$

are all distinct, explain why C actually corrects at least **2** errors.

[10 points]

(2) Let $C \subseteq \mathbb{F}_2^n$ be a linear code, and $\vec{v} \in \mathbb{F}_2^n$ where $\vec{v} \notin C$.

If $C + \vec{v} = \{\vec{c} + \vec{v} \mid \vec{c} \in C\}$ show that $C' = C \cup (C + \vec{v})$ is also a linear code.

[10 points]