

Radon–Nikodym Theorem for Signed Measures. *Let (X, \mathcal{E}, μ) be sigma finite and suppose ν is a signed measure on \mathcal{E} such that $\nu \ll \mu$. Then there exists a measurable function f such that $\int f d\mu$ is meaningful and $\nu(E) = \int_E f d\mu$ for every $E \in \mathcal{E}$. Also*

$$|\nu|(E) = \int_E |f| d\mu, \nu^+(E) = \int_E f^+ d\mu, \nu^-(E) = \int_E f^- d\mu \quad \forall E \in \mathcal{E}.$$

The function f is “unique a.e.” in the sense used in the Radon–Nikodym Theorem for measures.