

Let \mathcal{C} be a semialgebra of subsets of a nonempty set Z , and suppose that λ is a nonnegative set function, which is defined on the $C \in \mathcal{C}$ and is “finitely additive on \mathcal{C} ” and “countably subadditive on \mathcal{C} ” as defined in class. Let $\mathcal{A} = \mathcal{A}(\mathcal{C})$ be the algebra generated by \mathcal{C} , which consists of all finite disjoint unions of sets from \mathcal{C} . As we saw λ may be extended to \mathcal{A} by setting $\lambda(A) = \sum_{i=1}^n \lambda(C_i)$, where $A = \bigcup_{i=1}^n C_i$ is any representation of A as a finite disjoint union of sets $C_i \in \mathcal{C}$. We showed that λ is monotonic on \mathcal{A} . You are to show that λ is countably additive on \mathcal{A} , i.e., if $A \in \mathcal{A}$ is such that $A = \bigcup_{n=1}^{\infty} A_n$, where the $A_n \in \mathcal{A}$ and are pairwise disjoint, then $\lambda(A) = \sum_{n=1}^{\infty} \lambda(A_n)$. You might try proving this by first showing that λ is finitely additive on \mathcal{A} and countably subadditive on \mathcal{A} . If you do this you must also show how countable additivity then follows.