Problems

1. Show that if $\varphi = c_1\chi_{E_1} + c_2\chi_{E_2}$, where $c_1, c_2 > 0$ and E_1, E_2 are measurable but not necessarily disjoint, we still have

$$\int_E \varphi = c_1 m(E_1 \cap E) + c_2 m(E_2 \cap E).$$

Prove using the Lemma at the link above, which covers the case in which E_1 and E_2 are disjoint.

2. Prove, using the definitions of integrals *as given in lecture, not in Royden* that if $f \geq 0$ is defined and measurable on $B \in \mathcal{M}$ and if $A \subset B$ is measurable, then $\int_A f \leq \int_B f$.

3. Prove, using the definitions of integrals *as given in lecture, not in Royden*, that if $f \geq 0$ is measurable on $E \in \mathcal{M}$ and $a \in \mathbb{R}$ is nonnegative then $\int_E af = a \int_E f$.