1 Absolute Moments Method

Consider the aggregated series (1.1), obtained by dividing a given series (assumed to be FGN) of length N into blocks of length m, and averaging the series over each block.

$$X^{(m)}(k) := \frac{1}{m} \sum_{i=(k-1)m+1}^{km} X_i, \quad k = 1, 2, ..., [N/m]. \tag{1.1}$$

We take the n^{th} absolute moment of this series,

$$AM_n^{(m)} = \frac{1}{N/m} \sum_{k=1}^{N/m} \left| X^{(m)}(k) - \overline{X} \right|^n, \tag{1.2}$$

where $X^{(m)}$ is defined in (1.1) and $\overline{X}$ is the overall series mean. The aggregated series $X^{(m)}$ asymptotically behaves like $Cm^{n(H-1)}$ for large m, and thus $AM_n^{(m)}$ is proportional to $m^{n(H-1)}$.

This method is typically used for $n = 1$ (absolute mean). For $n = 2$, it reduces to the Aggregated Variance method.

For successive values of m, the sample absolute moment of the aggregated series is plotted versus m on a log-log plot. The result should be a straight line with a slope of $n(H-1)$. In practice, the slope is estimated by fitting a least-squares line to the points of the plot. We assume that both N and N/m are large. This ensures that both the length of each block and the number of blocks is large. If the series has no long-range dependence and finite variance, then $H = 0.5$ and the slope of the fitted line should be $-n/2$. In practice, the points at the very low and high ends of the plots are not used to fit the least-squares line. Indeed, short-range effects can distort the estimates of H if the low end of the plot is used, and at the very high end of the plot there are too few blocks to get reliable estimates of AM_n.

2 Fractal Dimension Method ($n = 1$)

A technique suggested by Higuchi [1] is very similar to the Absolute Moments method with $n = 1$. It uses,

$$L(m) = \frac{N-1}{m^3} \sum_{i=1}^{m} \left[\frac{N-1}{m}\right]^{-1} \sum_{k=1}^{[(N-i)/m]} \left| \sum_{j=i+(k-1)m+1}^{i+km} X(j) \right|. \tag{2.1}$$

Here, N is the length of the time series, m is the equivalent of a block size, and [ ] denotes the greatest integer function. For long-range dependent series, $E(L(m)) \sim Cm^{H-2}$. Essentially, the difference with the absolute moments method ($n = 1$) lies in using a sliding window to compute the aggregated series, instead of using non-intersecting blocks. Because of this, the method is much more computationally intensive. While this modification may result in increased accuracy in shorter time series, there seems to be no advantage in using it for the series length of $10,000$. This technique corresponds to the file *plotfract.S*.

Both of the techniques are discussed in Taqqu, Teverovsky and Willinger [3] and Taqqu and Teverovsky [2]. For an application to multifractals, see [4].

References

[1] T. Higuchi. Approach to an irregular time series on the basis of the fractal theory. *Physica D*, 31:277–283, 1988.

[2] M. S. Taqqu and V. Teverovsky. Estimating long-range dependence in finite and infinite variance series. In R. Adler, R. Feldman, and M. S. Taqqu, editors, *A Practical Guide to Heavy Tails: Statistical Techniques for Analyzing Heavy-Tailed Distributions*, Boston, 1996. Birkhäuser. To appear in 1997.

[3] M. S. Taqqu, V. Teverovsky, and W. Willinger. Estimators for long-range dependence: an empirical study. *Fractals*, 3(4):785–798, 1995. Reprinted in *Fractal Geometry and Analysis*, C.J.G. Evertsz, H-O Peitgen and R.F. Voss, editors. World Scientific Publishing Co., Singapore, 1996.

[4] M. S. Taqqu, V. Teverovsky, and W. Willinger. Is network traffic self-similar or multifractal? *Fractals*, 1996. To appear.