1 Variance of Residuals Method

The Variance of Residuals method was introduced by Peng et al. [1]. First the series is divided into blocks of size m. Then, within each block, the partial sums of the series are calculated, $Y(t) = \sum_{i=1}^{t} X_i$. A least-squares line, $a + bt$, is fitted to the partial sums within each block, and the sample variance of the residuals is computed,

$$\frac{1}{m} \sum_{t=1}^{m} \left(Y(t) - a - bt\right)^2 .$$

The variance of residuals is proportional to m^{2H}. For a proof in the Gaussian case, see [3], and for a sketch of the proof in other cases see [2].

This variance of residuals is computed for each block, and the median (or average) is computed over the blocks. A log-log plot versus m should follow a straight line with a slope of $2H$.

References

[1] C. K. Peng, S. V. Buldyrev, M. Simons, H. E. Stanley, and A. L. Goldberger. Mosaic organization of DNA nucleotides. *Physical Review E*, 49:1685–1689, 1994.

[2] M. S. Taqqu and V. Teverovsky. Estimating long-range dependence in finite and infinite variance series. In R. Adler, R. Feldman, and M. S. Taqqu, editors, *A Practical Guide to Heavy Tails: Statistical Techniques for Analyzing Heavy-Tailed Distributions*, Boston, 1996. Birkhäuser. To appear in 1997.

[3] M. S. Taqqu, V. Teverovsky, and W. Willinger. Estimators for long-range dependence: an empirical study. *Fractals*, 3(4):785–798, 1995. Reprinted in *Fractal Geometry and Analysis*, C.J.G. Evertsz, H-O Peitgen and R.F. Voss, editors. World Scientific Publishing Co., Singapore, 1996.