1 Whittle Method

The Whittle estimator is the value of the vector η which minimizes the function

$$Q(\eta) := \int_{-\pi}^{\pi} \frac{I(\nu)}{f(\nu;\eta)} d\nu + \int_{-\pi}^{\pi} \log f(\nu;\eta) d\nu. \tag{1.1}$$

The term $\int_{-\pi}^{\pi} \log f(\nu;\eta) d\nu$ can be set equal to 0 by renormalizing $f(\nu;\eta)$. The normalization only depends on a scale parameter, and not on the rest of the components of η. Thus, we replace f by f^*, such that $f^* = \beta f$, and $\int_{-\pi}^{\pi} \log f^*(\nu;\eta) d\nu = 0$. The first term in (1.1) stays unchanged except for division by β. The parameter β is regarded as an extra parameter, which we do not care to estimate. Since β does not depend on the other parameters, it will not affect the minimization of Q over these parameters. In actual applications, instead of an integral, the corresponding sum over the Fourier frequencies, $\nu_j = 2\pi j/N$ is computed, where $j = 1, 2, \ldots [(N-1)/2]$ and N is the length of the series. Thus, the actual function which the algorithm minimizes is:

$$Q^*(\eta) = \sum_{j=1}^{[(N-1)/2]} \frac{I(\nu_j)}{f^*(\nu_j;\eta)}. \tag{1.2}$$

When dealing with fractional Gaussian noise or FARIMA$(0, d, 0)$, η is the parameter H or d respectively. If the series is assumed to be FARIMA(p, d, q) then η also includes the unknown coefficients in the autoregressive and moving average parts. For such series the Whittle estimate $\hat{H}$ of H converges to its true value at the rate of $N^{1/2}$ and the asymptotic distribution of $\sqrt{N}(\hat{H} - H)$ is Gaussian. For details see Fox and Taqqu [1].

1.1 Aggregated Whittle

The aggregated Whittle estimator provides an additional technique for obtaining a robust estimate of H. It can be used if the time series is long enough. The idea is to aggregate the data, which creates a new, shorter series

$$X_i^{(m)} := \frac{1}{m} \sum_{j=m(i-1)+1}^{mi} X_j. \tag{1.3}$$

The shorter series increases the standard deviation of the estimator. On the other hand, however, if the aggregation level m is high enough and long-range dependence is present, then the new series will be close to Fractional Gaussian Noise. Then, the Whittle estimator with an underlying FGN model can be used on the series.

For details on these estimators see [2], [3],

References

[1] R. Fox and M. S. Taqqu. Large-sample properties of parameter estimates for strongly dependent stationary Gaussian time series. *The Annals of Statistics*, 14:517–532, 1986.

[2] M. S. Taqqu and V. Teverovsky. Robustness of Whittle-type estimates for time series with long-range dependence. Preprint, 1995.

[3] M. S. Taqqu and V. Teverovsky. Semi-parametric graphical estimation techniques for long-memory data. In P. M. Robinson and M. Rosenblatt, editors, *Athens Conference on Applied Probability and Time Series Analysis. Volume II: Time Series Analysis in Memory of E. J. Hannan*, pages 420–432, New York, 1996. Springer-Verlag. Lecture Notes in Statistics, **115**.