BUNTES FALL 2024: AUTOMORPHIC REPRESENTATIONS

1. References

The following two are the main references.

[G] Getz, J. R. *An introduction to automorphic representations*. Lecture notes. Available at https://services.math.duke.edu/~jgetz/aut_reps.pdf

[GH] Getz, J. R., & Hahn, H. (2024). *An Introduction to Automorphic Representations: With a view toward trace formulae*. Graduate Texts in Mathematics vol. 300. Springer.

Other references include:

(1) Bump, D. (1997). *Automorphic Forms and Representations*. Cambridge: Cambridge University Press.

(2) Bushnell, C. J., & Henniart, G. (2006). *The Local Langlands Conjecture for* $\mathrm{GL}(2)$. Grundlehren der mathematischen Wissenschaften vol. 335. Springer Berlin Heidelberg.

(3) Jacquet, H., & Langlands, R. P. (1970). *Automorphic Forms on* $\mathrm{GL}(2)$*: Part 1*. LNM vol. 114. Springer Berlin Heidelberg.

(4) Brian Conrad's notes on adelic points. Available at https://math.stanford.edu/~conrad/papers/adelictop.pdf

2. Schedule

Time: Wed 11:00 AM - 12:00 PM
Location: CDS 365

Week	Date	Topic
Week 1	Sep 11	Overview and affine group schemes
Week 2	Sep 18	Automorphic representations
Week 3	Sep 25	Non-archimedean representations
Week 4	Oct 2	Archimedean representations
Week 5	Oct 9	Automorphic forms
Week 6	Oct 16	Flath's theorem
Week 7	Oct 23	Cuspidal and supercuspidal representations
Week 8	Oct 30	Unramified representations
Week 9	Nov 6	Rankin-Selberg L-functions
Week 10	Nov 13	Simple trace formulas I

Week	Date	Topic
Week 11	Nov 20	Simple trace formulas II
Week 12	Nov 27	No Meeting: Thanksgiving
Week 13	Dec 4	Relative trace formulas
Week 14	Dec 11	TBD

3. Descriptions of the topics

Week 1: Overview and affine group schemes. Summary of basic ideas and applications of automorphic representations. Review of affine group schemes and their adelic points.

Week 2: Automorphic representations. Hilbert space and unitary representations of locally compact groups. Haar measures. Gelfand-Pettis integrals.Discrete automorphic representations. Ref: [GH] Chapter 3, [G] Section 3.

Week 3: Non-archimedean representations. Smooth and admissible representations. Hecke algebras. Contragredients. Traces, characters, coefficients. Ref: [GH], Chapter 8 [G] Section 4

Week 4: Archimdean representations. Smooth and admissible representations. $(\mathfrak{g}, K)$-modules. Hecke algebras. Ref: Ref: [GH] Chapter 4, [G] Section 5

Week 5: Automorphic forms. Definitions of automorphic forms. Examples. From modular forms to automorphic forms. Casimir elements. Ref: [GH] Chapter 6, [G] Section 6

Week 6: Flath's theorem. Factorizations of automorphic representations: Flath's theorem and its proof. Ref: [GH] Chapter 5, [G] Sections 7,8.

Week 7: Cuspidal and supercuspidal representations. Definitions of cuspidal and supercuspidal representations. Jacquet module. Discreteness of the cuspidal spectrum. Ref: [GH] Chapters 8,9, [G] Section 11. (This may need a second talk)

Week 8: Unramified representations. Satake isomorphism. Ramanujan and Ramanujan-Petersson conjectures. Ref: [GH] Chapters 7, [G] Section 9.

Week 9: Rankin-Selberg L-functions.

Week 10: Simple trace formulas I. Traces, kernel functions, orbital integrals.

Week 11: Simple trace formulas II. Some trace formulas, applications.

Week 13: Relative trace formulas.

Possible additional topics: Eisenstein series, cohomology of locally symmetric spaces, Langlands conjectures and functoriality.