

INTRODUCTION TO PRISMATIC COHOMOLOGY I

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1. HISTORICAL MOTIVATIONS

Recall the A_{inf} -cohomology constructed in [BMS18] and extended to the semistable case in [CK19]. Let C be a complete algebraically nonarchimedean field over \mathbb{Q}_p with residue field k , and let \mathfrak{X} be a smooth proper scheme over $\text{Spf } \mathcal{O}_C$. Write X for the adic generic fiber of \mathfrak{X} . Then we have a complex $A\Omega_{\mathfrak{X}} \in D(\mathfrak{X}_{\text{Zar}})$ given by

$$A\Omega_{\mathfrak{X}} := L\eta_{\mu}(R\nu_* \mathbb{A}_{\text{inf}, X}),$$

where $\nu : X_{\text{proét}} \rightarrow X_{\text{ét}}$ is the canonical projection, $\mathbb{A}_{\text{inf}, X} := W(\mathcal{O}_X^{\flat+})$ is the usual infinitesimal period sheaf, and $L\eta_{\mu}$ is the décalage functor with respect to the distinguished element $\mu \in A_{\text{inf}} = \mathbb{A}_{\text{inf}}(\mathcal{O}_{C^b})$. The A_{inf} -cohomology $R\Gamma_{A_{\text{inf}}}(\mathfrak{X})$ of \mathfrak{X} is then defined as

$$R\Gamma_{A_{\text{inf}}}(\mathfrak{X}) := R\Gamma(\mathfrak{X}_{\text{Zar}}, A\Omega_{\mathfrak{X}}).$$

There are a few comparison results between $R\Gamma_{A_{\text{inf}}}(\mathfrak{X})$ and the other well-known p -adic cohomology theories.

Theorem 1.1 ([BMS18] Theorem 1.10). *As above, assume \mathfrak{X} is proper smooth.*

(1) *Crystalline comparison: Let \mathfrak{X}_k denote the (reduced) special fiber of \mathfrak{X} . Then*

$$A\Omega_{\mathfrak{X}} \widehat{\otimes}_{A_{\text{inf}}}^{\mathbb{L}} W(k) \simeq W\Omega_{\mathfrak{X}_k/W(k)}^{\bullet}.$$

(2) *de Rham comparison:*

$$A\Omega_{\mathfrak{X}} \otimes_{A_{\text{inf}}}^{\mathbb{L}} \mathcal{O}_C \simeq \Omega_{\mathfrak{X}/\mathcal{O}_C}^{\bullet, \text{cont}},$$

where $\Omega_{\mathfrak{X}/\mathcal{O}_C}^{i, \text{cont}} = \varprojlim_n \Omega_{(\mathfrak{X}/p^n)/(\mathcal{O}_C/p^n)}^i$.

(3) *Etale comparison:*

$$A\Omega_{\mathfrak{X}} \otimes_{A_{\text{inf}}} A_{\text{inf}}[1/\mu] \simeq (R\nu_* \mathbb{A}_{\text{inf}, X}) \otimes_{A_{\text{inf}}} A_{\text{inf}}[1/\mu].$$

However, it seems very hard to extend the results to non-smooth or non-proper cases. The semistable proper case, which is the mildest non-smooth case, is treated in [CK19], but the authors have to do many complicated local computations to get the results. The other drawback of the A_{inf} -cohomology is that there is no hope to have analogous results over bases like \mathbb{Z}_p if one works within the framework of the A_{inf} -cohomology. This is because the base need to have all p -power roots of unity to have the distinguished element μ .

The rescue of all these problems is the prismatic cohomology in [BS22]. For a p -adic formal scheme X over a prism A (to be defined later), the (relative) prismatic cohomology $\Delta_{X/A}$, roughly speaking, is the cohomology of the structure sheaf \mathcal{O}_{Δ} on the relative prismatic site $(X/A)_{\Delta}$. Modulo some technical conditions, we have comparisons between the prismatic cohomology $\Delta_{X/A}$ to all well-known p -adic cohomology theories, and the p -adic formal scheme is not required to be smooth or proper. The prismatic cohomology also allows a natural coefficient systems, namely, vector bundles (or perfect complexes) over the structure sheaf \mathcal{O}_{Δ} . This flexibility in coefficients gives rise to the classification of crystalline representations and local systems in [BS23] and [GR24].

Let us summarize the main comparison results from [BS22] now.

Theorem 1.2. *Let (A, I) be a bounded prism, and let X be a smooth p -adic formal scheme over $\text{Spf } A/I$, where A/I has the p -adic topology.*

- (1) *Crystalline comparison: If $I = (p)$, then there is a canonical ϕ -equivariant isomorphism*

$$R\Gamma_{\text{crys}}(X/A) \simeq R\Gamma_{\Delta}(X/A) \widehat{\otimes}_{A, \phi_A}^L A = \phi_A^* R\Gamma_{\Delta}(X/A)$$

of commutative algebras in $D(A)$.

- (2) *Hodge-Tate comparison: If $X = \text{Spf } R$ is affine, there is a canonical R -module isomorphism*

$$\Omega_{R/(A/I)}^i \{-i\} \simeq H^i(R\Gamma_{\Delta}(X/A) \otimes_A^L A/I).$$

Here for any A/I -module M , $M\{i\} := M \otimes_{A/I} (I/I^2)^{\otimes i}$ is the i -th Breuil-Kisin twist.

- (3) *de Rham comparison: There is a canonical isomorphism*

$$R\Gamma_{\text{dR}}(X/(A/I)) \simeq R\Gamma_{\Delta}(X/A) \widehat{\otimes}_{A, \phi_A}^L A/I = \phi_A^* R\Gamma_{\Delta}(X/A) \otimes_A^L A/I$$

of commutative differential graded algebras in $D(A)$.

- (4) *Etale comparison: Assume A is perfect. Let X_{η} be the adic generic fiber of X over \mathbb{Q}_p treated as a diamond. For any $n \geq 0$, there is a canonical isomorphism*

$$R\Gamma_{\text{et}}(X_{\eta}, \mathbb{Z}/p^n \mathbb{Z}) \simeq (R\Gamma_{\Delta}(X/A) \otimes_A^L (A/p^n)[1/I])^{\phi=1}$$

of commutative algebras in $D(\mathbb{Z}/p^n)$

- (5) *A_{inf} comparison: Let C, \mathcal{O}_C be as above, and let $A := A_{\text{inf}} = A_{\text{inf}}(\mathcal{O}_C)$ with $I := \ker(\theta : A_{\text{inf}} \rightarrow \mathcal{O}_C)$. Then there exists a canonical ϕ -equivariant isomorphism*

$$R\Gamma_{A_{\text{inf}}}(X) \simeq \phi_A^* R\Gamma_{\Delta}(X/A_{\text{inf}}).$$

The generality of the results for the prismatic cohomology does come with some costs. One of the issues is one always needs to use homotopical algebra, such as derived completions, descent, and

∞ -categories. But fortunately, as we will see, most of the issues can be addressed very elegantly. So there is no need to be scared by some unfamiliar words in this theory.

The plan of these notes is as follows: we will introduce the basics of the prismatic cohomology based on [BS22], [BL22] and many other expository resources. In particular, we will go over the main comparison theorems in even non-smooth case. After that, we will discuss the proof of a very general form of Fontaine's C_{crys} -conjecture in [GR24]. We hope this can give people some sense on what the prismatic cohomology can do.

2. δ -RINGS

The prismatic cohomology is formulated based on δ -rings, which are roughly rings equipped with lifts of the Frobenius. In fact, the central objects, prisms, are δ -rings with additional structures and properties.

Definition 2.1. A δ -ring is a pair (A, δ) consisting of a ring A and a map of sets $\delta : A \rightarrow A$ satisfying: for any $x, y \in A$

$$\delta(0) = \delta(1) = 0$$

$$\delta(xy) = x^p \delta(y) + y^p \delta(x) + p\delta(x)\delta(y)$$

$$\delta(x + y) = \delta(x) + \delta(y) - \sum_{i=1}^{p-1} \frac{(p-1)!}{i!(p-i)!} x^i y^{p-i} = \delta(x) + \delta(y) - \frac{x^p + y^p - (x+y)^p}{p}.$$

We will call such a map δ a p -derivation.

We also define map $\phi : A \rightarrow A$ by

$$\phi(x) = x^p + p\delta(x).$$

The requirements on the map δ are meant to make ϕ a lift of Frobenius.. This is made precise by the following lemma.

Lemma 2.2.

- (1) *The map $\phi : A \rightarrow A$ is a ring homomorphism that induces the Frobenius map $x \mapsto x^p$ on A/pA .*
- (2) *If A is p -torsion-free, then this construction gives a bijection between p -derivations δ and Frobenius lifts on A .*

Proof. Proof of (1) is straightforward. For (2), as A is p -torsion-free, the formula $\phi(f) = f^p + p\delta(f)$ uniquely defines $\delta(f)$ given ϕ . So it suffices to show that δ so defined satisfies the condition in the definition of δ -rings, which is easy. \square

Example 2.3.

- (1) Maybe the most important examples of δ -rings are the rings of Witt vectors. So suppose R is a perfect \mathbb{F}_p -algebra. Then the ring of Witt vectors $W(R)$ admits a unique lift of Frobenius, which makes $W(R)$ a δ -ring. This also shows $W(R)$ has a unique δ -structure.
- (2) The ring \mathbb{Z} with $\delta(n) = \frac{n-n^p}{p}$ is a δ -ring. This makes ϕ the identity map. In fact (\mathbb{Z}, δ) is the initial object in the category of δ -rings.

- (3) If A is a $\mathbb{Z}[1/p]$ -algebra, then any endomorphism ϕ of A is a δ -structure since the condition that ϕ lifts the Frobenius on A/p is vacuously true.
- (4) The free δ -ring $\mathbb{Z}\{x\}$ on a variable x is the polynomial ring $\mathbb{Z}[x_0, x_1, x_2, \dots]$ with $x = x_0$ and $\delta(x_i) = x_{i+1}$. In general, for any set S , we can form the free δ -ring $\mathbb{Z}\{S\}$ on S .

Definition 2.4. A morphism of δ -rings $f : (A, \delta) \rightarrow (A', \delta')$ is a ring homomorphism $f : A \rightarrow A'$ such that $f \circ \delta = \delta' \circ f$.

The category of δ -rings is denoted as **Ring** $_{\delta}$.

Next we demonstrate a connection between δ -structures and the length-2 Witt vectors W_2 . For any ring A , $W_2(A)$ can be defined explicitly as follows: $W_2(A) = A \times A$ as sets, and addition and multiplication are defined by

$$(x, y) + (z, w) := (x + z, y + w + \frac{x^p + z^p - (x + z)^p}{p})$$

and

$$(x, y) \cdot (z, w) = (xz, x^p w + z^p y + pyw).$$

We have a natural projection homomorphism $\epsilon : W_2(A) \rightarrow A, (x, y) \mapsto x$. It is immediate from the definition that a δ -structure on A is the same as a ring map $w : A \rightarrow W_2(A)$ such that $\epsilon \circ w = \text{id}_A$. More precisely, the correspondence is given by the relation $w(x) = (x, \delta(x))$.

Lemma 2.5. *The category Ring_{δ} admits arbitrary limits and colimits. The formations of limits and colimits commute with the forgetful functor to **Ring**.*

Proof. The limit of a diagram $\{A_i\}$ of δ -rings is given by the limit $\lim_i A_i$ of the underlying rings; one checks easily this gives a natural δ -structure on $\lim_i A_i$. For colimits, we use the W_2 -construction of δ -structures discussed above. For a diagram $\{A_i\}$ of δ -ring, let $w_i : A_i \rightarrow W_2(A_i)$ be the ring maps corresponding to the δ -structures. Taking colimits gives $\text{colim}_i A_i \rightarrow \text{colim}_i W_2(A_i)$. The functoriality of W_2 gives a natural map $\text{colim}_i W_2(A_i) \rightarrow W_2(\text{colim}_i A_i)$. Composing the two maps gives a map $w : \text{colim}_i A_i \rightarrow W_2(\text{colim}_i A_i)$ whose composition with the projection to the first factor $W_2(\text{colim}_i A_i) \rightarrow \text{colim}_i A_i$ is the identity. This gives a δ -structure on $\text{colim}_i A_i$, which completes the construction of the colimit of $\{A_i\}$ as δ -rings. \square

By general category theory, the forgetful functor **Ring** $_{\delta} \rightarrow \mathbf{Ring}$ admits both a left adjoint and a right adjoint. The right adjoint is the Witt vectors. For any set S , applying the left adjoint functor to the polynomial ring $\mathbb{Z}[S]$ gives the free δ -ring $\mathbb{Z}\{S\}$.

3. DERIVED COMPLETIONS

Before we introduce the central concept, prisms, we need to address some concepts in derived algebra that are necessary to the prismatic cohomology.

Definition 3.1. Let A be a ring, and I a finitely generated ideal of A . An A -module M is classically I -complete if the natural map $M \rightarrow \lim_n M/I^n M$ is an isomorphism.

Note this in particular means M is I -adically separated, i.e., $\bigcap_n I^n M = 0$.

We mention a few issues with the notion of (classical) completion.

- (1) Classically complete A -modules do not form an abelian category.
- (2) completion is neither right nor left exact in general. In fact, completion can be viewed as the composition of tensor product, which is right exact, and inverse limit, which is left exact.
- (3) The completion of a flat module may not be flat.
- (4) If I is not required to be finitely generated, then the I -adic completion $\lim_n M/I^n M$ of a module M may not be complete.

The point is that classical completion is not robust enough. We need a weaker but more flexible notion of completion.

Definition 3.2. Let A be a ring, and I a finitely generated ideal of A . An A -module M is derived I -complete if for $f \in I$,

$$\mathrm{Hom}_A(A_f, M) = 0 \quad \text{and} \quad \mathrm{Ext}_A^1(A_f, M) = 0.$$

Remark 3.3. Note A_f admits a free resolution as an A -module:

$$0 \rightarrow A[T] \xrightarrow{\times(1-Tf)} A[T] \xrightarrow{T \mapsto f^{-1}} A_f \rightarrow 0.$$

So for any A -module M , $\mathrm{Ext}_A^n(A_f, M) = 0, \forall n \geq 2$. The condition in the definition above can be then reformulated as

$$\mathrm{Ext}_A^n(A_f, M) = 0, \forall n \geq 0.$$

Lemma 3.4.

- (1) If M is classically I -complete, then it is derived I -complete.
- (2) If M is derived I -complete, then the natural map $M \rightarrow \lim_n M/I^n M$ is surjective.

Proof of (1). Suppose M is classically I -complete. We have

$$\mathrm{Hom}_A(A_f, M) = \mathrm{Hom}_A(A_f, \lim_n M/I^n M) = 0.$$

To show that $\mathrm{Ext}_A^1(A_f, M) = 0$, consider an extension

$$0 \rightarrow M \rightarrow E \rightarrow A_f \rightarrow 0.$$

For each $n \geq 0$, pick $e_n \in E$ mapping to $f^{-n} \in A_f$, and set $\delta_n = fe_{n+1} - e_n \in M$. Since M is complete, we may define the elements

$$e'_n = e_n + \delta_n + f\delta_{n+1} + f^2\delta_{n+2} + \cdots$$

which satisfy $fe'_{n+1} = e'_n$. We thus obtain a map $A_f \rightarrow E$ splitting the sequence by mapping f^{-n} to e'_n . \square

Corollary 3.5. *Assumptions as above. M is classically I -complete if and only if M is I -adically separated and derived I -complete.*

Proof. Directly by Lemma 3.4. \square

Next, we introduce one of the most important results for derived complete modules, the derived Nakayama.

Lemma 3.6 (Derived Nakayama). *Let M be a derived I -complete A -module. Then $M = 0$ if and only if $M/IM = 0$.*

Proof. [Sta25, Tag 0G1U] □

The inclusion of the category of derived I -complete modules to the category of A -modules admits a left adjoint $M \mapsto \widehat{M}$, called derived I -completion. The full subcategory of A -modules consisting of derived I -complete modules form an abelian category.

We conclude this section by showing an explicit construction of derived I -completion in a simple case.

Example 3.7. Let $f \in A$ be a non-zerodivisor. The derived f -adic completion of M is given by

$$\widehat{M} = \text{Ext}_A^1(A/fA, M)[1].$$

So if $A = \mathbb{Z}$, $I = (p)$, and $M = \mathbb{Q}/\mathbb{Z}$, then $\widehat{M} := \mathbb{Z}_p[1]$

4. PRISMS

Fix a prime p .

Definition 4.1. A prism is a pair (A, I) of a δ -ring A and an invertible ideal I of A such that

- (1) A is derived (p, I) -complete.
- (2) $p \in (I, \phi(I))$ (equivalent to that after some localization of A , we have $I = (d)$ with $\delta(d)$ invertible in A).

We now define morphisms between prisms. But we first recall the following definition.

Definition 4.2. For a ring A with a finitely generated ideal I , an A -module M is I -completely flat (resp. I -completely faithfully flat) if M/IM is a flat (resp. faithfully flat) A/I -module and $\text{Tor}_i^A(A/I, M) = 0$ for $i > 0$. The last condition is equivalent to that for any A/I -module N , $\text{Tor}_i^A(N, M) = 0$ for $i > 0$. If M is (faithfully) flat, then it is I -completely (faithfully) flat.

Definition 4.3. A map $f : (A, I) \rightarrow (B, J)$ of prisms is a map of δ -rings $f : A \rightarrow B$ such that $f(I) \subseteq J$. A map $(A, I) \rightarrow (B, J)$ of prisms is called (faithfully) flat if the underlying ring map $A \rightarrow B$ is (p, I) -completely (faithfully) flat.

Morphisms of prisms have the following rigidity property.

Lemma 4.4. *If $f : (A, I) \rightarrow (B, J)$ is a map of prisms, then f induces an isomorphism $I \otimes_A B \simeq J$. In particular, $IB = J$.*

Conversely, if (A, I) is a prism and $A \rightarrow B$ is a map of δ -rings with B being derived (p, I) -complete, then (B, IB) is a prism exactly when $B[I] = 0$.

Example 4.5.

- (1) (Crystalline prism) Let A be a (classically) p -complete p -torsion-free δ -ring with $I = (p)$. Then $(A, (p))$ forms a prism, usually called a *crystalline prism*. Note in this case, $\delta(p) = \frac{p-p^p}{p} = 1 - p^{p-1}$ is invertible in A .

- (2) (Breuil-Kisin prism) Suppose k is a perfect field of characteristic p . Let $A = W(k)[[u]]$ with $\delta(u) = 0$ (equivalent to $\phi(u) = u^p$) and $I = (E(u))$, where $E(u)$ is an Eisenstein polynomial. Note $A/I \simeq \mathcal{O}_K$ via map $u \mapsto \pi$, where \mathcal{O}_K is the ring of integers of a p -adic field K totally ramified over $W(k)[1/p]$, and π is a uniformizer in K . A prism of the form $(W(k)[[u]], (E(u)))$ is called a *Breuil-Kisin prism*.
- (3) (q -crystalline prism) Let $A = \mathbb{Z}_p[[q-1]]$, which is the $(p, [p]_q)$ -adic completion of $\mathbb{Z}[q]$. Let $I = ([p]_q)$, where

$$[p]_q = \frac{q^p - 1}{q - 1} = 1 + q + \cdots + q^{p-1}$$

is the q -analog of p . Then (A, I) is a prism.

- (4) (δ -ring with no prism structure) A δ -ring can have many prism structures, i.e., many choices of I . But there are also δ -rings with no prism structure. Consider the local ring $W(\mathbb{F}_p[x]/(x^2))$ complete with respect to the p -adic topology. Its δ -structure comes from the W_2 -construction discussed above. The ring $W(\mathbb{F}_p[x]/(x^2))$ does not have any prism structure: any non-zero-divisor d of $W(\mathbb{F}_p[x]/(x^2))$ is a unit.

The names of the prisms in the example come from their appearance in the classical theory or comparison theorems with the prismatic cohomology. For example, crystalline prisms are those over which we can formulate the crystalline comparison, and the Breuil-Kisin prisms already appear in the classical theory of Breuil-Kisin modules.

Definition 4.6. A prism (A, I) is

- (1) bounded if A/I has bounded p^∞ -torsions. This implies A is classically (p, I) -complete.
- (2) orientable if I is principal. A choice of the generator d of I is called an orientation.
- (3) perfect if $\phi : A \rightarrow A$ is an isomorphism.
- (4) transversal if A/I is p -torsion free.

The notions "bounded", "orientable", and "transversal" are mainly for technical reasons. We will always try to reduce the problems to these three cases so we have better algebra results to use. As shown below, perfect prisms are equivalent to perfectoid ring. We will also often try to reduce to this case so that we can use perfectoid geometry, such as the tilting equivalence. Note also the definition of perfect prisms is independent of the ideal I . Here we take the definition of perfectoid rings as in [BMS18, §3]

Definition 4.7. A ring S is perfectoid if and only if it is π -adically complete for some element $\pi \in S$ such that π^p divides p , the Frobenius map $\phi : S/pS \rightarrow S/pS$ is surjective, and the kernel of $\theta : \mathbb{A}_{\text{inf}}(S) \rightarrow S$ is principal.

Example 4.8. The following rings are examples of perfectoid algebras. First, any perfect \mathbb{F}_p -algebra is perfectoid (where we take $\pi = 0$); here, perfect means that the Frobenius map is an isomorphism. Moreover, the p -adic completion $\mathbb{Z}_p^{\text{cycl}}$ of $\mathbb{Z}_p[\zeta_{p^\infty}]$ is perfectoid; one may also take the p -adic completion of the ring of integers of any other algebraic extension of \mathbb{Q}_p containing the cyclotomic extension. Another example is $\mathbb{Z}_p^{\text{cycl}}\langle T^{1/p^\infty} \rangle$, and there are many obvious variants.

Remark 4.9. Note this definition is slightly more general than the one in [SW20], which has a stronger restriction on the topology on S . For example, the discrete ring \mathbb{F}_p is not considered as a perfectoid algebra in [SW20].

Lemma 4.10. *There is an equivalence of categories:*

- *The category of perfectoid rings R .*
- *The category of perfect prisms (A, I) .*

The functors are $R \mapsto (A_{\text{inf}}(R), \ker \theta)$ and $(A, I) \mapsto A/I$, respectively.

Proof. Let R be a perfectoid ring. Since $A_{\text{inf}}(R)$ is just the Witt vector construction on the tilt R^\flat , which is perfect and in characteristic p , $A_{\text{inf}}(R)$ is a perfect δ -ring, i.e., with a natural δ -structure and ϕ being an isomorphism. By the general results in p -adic Hodge theory, $A_{\text{inf}}(R)$ is also classically $(p, \ker \theta)$ -complete, and hence also derived $(p, \ker \theta)$ -complete. Thus, we get a prism.

Conversely, we claim the following properties of perfect prisms (A, I) :

- (1) I is principal and generated by a distinguished element d , i.e., $\delta(d)$ is a unit in A .
- (2) Any perfect prism (A, I) is bounded, and thus classically (p, I) -complete.

For a proof, see [BS22, Lemma 3.8] Since A is a perfect δ -ring, the Frobenius on R/p with $R := A/d = A/I$ is surjective. Since A is perfect, one can show $A \simeq W(S)$ for some perfect \mathbb{F}_p -algebra S . Via this isomorphism, we can write $d = [a_0] + pu$ for some unit $u \in A$. Let $\pi \in R$ be the image of $[a_0^{1/p}]$. Then $\pi^p | p$ in R . This shows R is perfectoid if R is also classically p -adically complete. \square

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