Rigid Geometry Seminar

The preliminary goal of the seminar is to cover the book "Lectures on Formal and Rigid Geometry" by S. Bosch. This book covers rigid geometry from two perspectives: the classical and the formal.

Let us say a little more about these two approaches. The goal of Part 1 is to develop algebraic geometry of rigid analytic spaces. Roughly speaking, a rigid space $(X, \mathscr{O}_X)$ is a locally ringed space built out of affinoid spaces (all with respect to a certain Grothendieck topology which gives an assignment of admissible opens and admissible covers of X). An affinoid space is the rigid analogue of an affine scheme in algebraic geometry, hence a rigid space is the rigid analogue of a scheme. As in algebraic geometry there is a sensible notion of coherent sheaves on rigid spaces (Ch. 6) which allows one to compute cohomology of rigid spaces.

Part 2 is on formal geometry, i.e. using formal schemes to understand rigid spaces. For simplicity set $R = \mathbf{Z}_p$ and $K = \mathbf{Q}_p$. There is a formal rigidification functor from the category of formal R-schemes to the category of rigid K-spaces. The rigid space we get from this construction is called the "Raynaud generic fiber" of the formal scheme. One may ask if we may realize every rigid space as the generic fiber of some formal scheme, i.e. whether every rigid space admits a formal model. This will be true for suitably nice rigid spaces, see [B14, Thm. 8.4.3]. This gives a direct connection between algebraic geometry and rigid geometry.

Finally it would be helpful to discuss applications/constructions in rigid geometry. Tate's original motivation for developing rigid analytic spaces was to provide a non-Archimedian analogue of complex uniformization of elliptic curves. This is described near the end of [B14] and in more detail in Ch. 5 of [FvdP]. Given additional time/energy at the end, [L16] generalizes the construction of the Tate elliptic curve to curves of higher genus (called "Mumford curves") and more. Another idea could be to compare the material in Bosch's book to the theory of adic spaces. If there are any other topics related to rigid geometry that you would like to see, please let me know and you can give a talk on it!

References

[B14] Bosch. *Lectures on Formal and Rigid Geometry*. https://link.springer.com/book/10.1007/978-3-319-04417-0

[FvdP] Fresnel, van der Put. *Rigid Analytic Geometry and Its Applications*. https://link.springer.com/book/10.1007/978-1-4612-0041-3

[L16] Lütkebohmert. *Rigid Geometry of Curves and Their Jacobians*. https://link.springer.com/book/10.1007/978-3-319-27371-6