

BOSTON UNIVERSITY NUMBER THEORY SEMINAR

# One-level density in one-parameter families of elliptic curves with non-zero average root number

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111 Cummington Street, MCS B21

Tea and cookies in MCS 144 at 4:00 pm

**Abstract:** We present in this talk a (conjectural) formula for the one-level density of general one-parameter families of elliptic curves, in terms of  $n$ , the rank  $E$  over  $\mathbf{Q}(t)$  and the average root number  $W_E$  over the family. In the general case,  $W_E$  is zero, and the one-level density is given by orthogonal symmetries as predicted by the conjectures of Katz and Sarnak. In the exceptional cases where  $W_E \neq 0$ , we find that the statistics are given by a weighted sum of even orthogonal and odd orthogonal symmetries. The most dramatic and counter-intuitive cases occur when  $W_E = \pm 1$ . In that case, the one-level density exhibits even orthogonal symmetries when  $(-1)^n W_E = 1$  and odd orthogonal symmetries when  $(-1)^n W_E = -1$ , and there is a shift of the symmetries (between orthogonal odd and orthogonal even) when  $n$  is odd.

We also build several one-parameter families of elliptic curves with  $W_E \neq 0$ , and which exhibits the shifts of the symmetries.