The Average Size of 2-Selmer Groups of Elliptic Curves over Function Fields

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Tea and cookies at 3:30

Abstract: Given an elliptic curve \( E \) over a global field \( K \), the abelian group \( E(K) \) is finitely generated, and so much effort has been put into trying to understand the behavior of rank \( E(K) \) as \( E \) varies. Of note, it is a folklore conjecture that, when all elliptic curves \( E/K \) are ordered by a suitably defined height, the average value of rank \( E(K) \) is exactly \( 1/2 \). One fruitful avenue for understanding the distribution of rank \( E(K) \) has been to first understand the distribution of the sizes of Selmer groups of elliptic curves. In this direction, various authors (including Bhargava-Shankar, Poonen-Rains, and Bhargava-Kane-Lenstra-Poonen-Rains) have made conjectures on how these Selmer groups are distributed, predicting, for example, that the average size of the \( n \)-Selmer group of \( E/K \) is equal to the sum of the divisors of \( n \). In this talk, I will report on some recent work verifying this average size prediction, "up to small error term," whenever \( n = 2 \) and \( K \) is any global *function* field. Results along these lines were previously known whenever \( K \) was a number field or function field of characteristic \( \geq 5 \), so the novelty of my work is that it applies even in "bad" characteristic.