

BOSTON UNIVERSITY NUMBER THEORY SEMINAR

On the growth of torsion in cohomology of arithmetic groups

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Monday, April 24 at 4:15 pm
111 Cummington Mall, MCS B21
Tea and cookies in MCS 144 at 4:00 pm

Abstract: Let G be a semisimple Lie group with associated symmetric space D , and let Γ be a cocompact arithmetic group of G . Bergeron and Venkatesh recently gave a precise conjecture about the growth of the order of the torsion subgroup $H_i(\Gamma_k)_{tors}$ as Γ_k ranges over a tower of congruence subgroups of Γ . In particular they conjectured that the ratio $(\log |H_i(\Gamma_k)_{tors}|)/([\Gamma : \Gamma_k])$ should tend to a nonzero limit if and only if $i = (\dim(D) - 1)/2$ and G is a group of so-called deficiency 1; such groups include for instance $SL_n(\mathbb{R})$ for $n = 3$ and 4. Furthermore, Bergeron and Venkatesh gave a precise expression for the limit.

In this talk, we describe our computational investigation of this phenomena. We consider the cohomology of several (non-cocompact) arithmetic groups, including $GL_n(\mathbb{Z})$ for $n=3,4,5$ and GL_2 over various rings of integers, and observe the growth of torsion as a function of level. In all cases where our dataset is sufficiently large, we observe excellent agreement with the same limit as in the predictions of Bergeron-Venkatesh. Our data also prompts us to make two new conjectures on the growth of torsion not covered by the Bergeron-Venkatesh conjecture. This is joint work with Avner Ash, Mark McConnell, and Dan Yasaki.