Thresholding and multi-body interactions orient cascades in spatially embedded networks

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Cascading phenomena over spatially embedded networks, such as social contagions, neuronal activations and infrastructure failures etc., exhibit two competing spreading mechanisms, local wavefront propagation (WFP) and nonlocal appearance of new clusters (ANC), which occur due to the presence of both short- and long-range edges. We are extending this field [1] to understand the effects of higher-order interactions on such cascades. We introduce a *simplicial threshold model* (STM) for analyzing such systems with dyadic, triadic and higher-order dependencies. We utilize topological building blocks, k-*simplex*, to encode the interactions of dimension k and simulate cascades in which vertex v_i gets activated if the surrounding activity of k-simplices exceeds a certain threshold, T. We show that higher-order interactions and thresholding cooperatively guide cascades along multidimensional geometrical channels. We study STM cascades on a C. Elegans "neuronal complex", which we construct using an empirical synapse network, showing that higher-order interactions enhance the memory capacity and efficiency of STM cascades. We support our findings with bifurcation theory to predict wavefront speeds and cluster appearance rates. Time permitting, I will talk about generalized STM models exhibiting non-monotonic behavior, which give rise to pulse-type wavefronts.

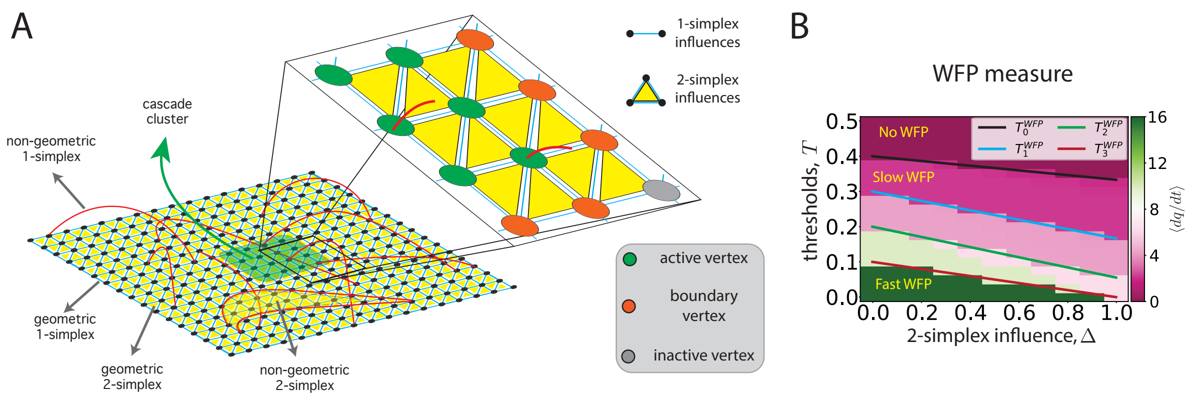

Figure 1: **Simplicial threshold model (STM) for higher-order cascades over a *noisy geometric complex*. (A)** A STM cascade initialized near the center of a 2D simplicial complex with both short- and long-range k-simplices. 1- and 2-simplices are visualized by edges and triangles, respectively. Propagation requires the aggregate activity across its simplicial neighbors—which includes adjacent 1-simplices, 2-simplices, etc.—to surpass a threshold T, and spreading across short-range k-simplices yields wavefront propagation (WFP), whereas propagation across long-range simplices causes the appearance of new clusters (ANC). **(B)** We let parameter $\Delta \in [0, 1]$ tune the relative influence of 1-simplices vs. 2-simplices and develop bifurcation theory to predict the rates of wavefront propagation (shown) and the appearance of new clusters for different T and Δ.

References

[1] Kilic, B. U. and Taylor, D. Simplicial cascades are orchestrated by the multidimensional geometry of neuronal complexes. Preprint. arXiv:2201.02071 (2022).