Spectral stability of the vortices for the NLS in higher dimensions

Wen Feng, Milena Stanislavova

Abstract: We consider the nonlinear Schrödinger equation in n space dimension

$$iu_t + \triangle u + |u|^{p-1}u = 0,\ x \in \mathbb{R}^n,\ t > 0$$

and study the existence and stability of standing wave solutions of the form

$$\begin{cases} e^{iwt}e^{i\sum_{j=1}^{k} m_j\theta_j}\phi_w(r_1, r_2, \cdots, r_k), & n = 2k \\ e^{iwt}e^{i\sum_{j=1}^{k} m_j\theta_j}\phi_w(r_1, r_2, \cdots, r_k, z), & n = 2k+1 \end{cases}$$

for $n = 2k$, (r_j, θ_j) are polar coordinates in $\mathbb{R}^2$, $j = 1, 2\cdots, k$; for $n = 2k+1$, (r_j, θ_j) are polar coordinates in $\mathbb{R}^2$, (r_k, θ_k, z) are cylindrical coordinates in $\mathbb{R}^3$, $j = 1, 2\cdots, k-1$. We show the existence of such positive solutions, which are constructed variationally as minimizers of appropriate constrained functionals. These waves are shown to stable (with respect with perturbations of the same type), if $1 < p < 1 + 4/n$.