Moduli of bundles on symplectic surfaces

Eyal Markman
Department of Mathematics
Amherst

Abstract

We study the cohomology ring of moduli spaces of vector bundles on holomorphic-symplectic surfaces. We first obtain generators for the ring in the case of

1) The moduli space of complex representations of the fundamental group of a Riemann-Surface (due, originally, to Hausel-Thaddeus),

2) Hilbert schemes of points on K3 and Abelian surfaces, and other moduli of vector bundles.

Next, we calculate the monodromy action on the cohomology of moduli of bundles over a K3 surface. The question is interesting, since the moduli, considered as a hyperkahler variety, admits deformations which do not come from a K3 surface. We use a higher dimensional symplectic analogue of the monodromy around a K3 with a double point. In the higher dimensional case, the singularity of moduli is no-longer isolated. It is modeled after a nilpotent orbit in sl(n).