There are enough Azumaya algebras on surfaces

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Abstract

Roughly speaking, an Azumaya algebra is a bundle of central simple algebras, and such bundles show up in various contexts. A notorious problem about Brauer groups and étale cohomology is whether each torsion class in $H^2(X_{et}, G_m)$ comes from an Azumaya algebra.

In this talk, I shall discuss what is known about this problem, and prove that for separated normal surfaces, the cohomological Brauer group indeed equals the geometric Brauer group. For this result, the idea is to apply elementary transformations, introduced by Maruyama for vector bundles, to Brauer–Severi schemes, and to relate endomorphisms of reflexive sheaves to Azumaya algebras.