Zooming in on the Hitchin system in genus 2

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Abstract

The Hitchin system of the title is an algebraically completely integrable system whose integral manifolds are Prymians of dimension $3g-3$, defined on the holomorphic-symplectic manifold $\mathcal{T}^*\mathcal{SU}_X(2,\xi)$, the cotangent bundle to the moduli space of vector bundles of rank 2 and fixed odd determinant ξ, over a Riemann surface X of genus $g>1$. Further work by Hitchin (1990) implemented geometric quantization and provided a link of the system with the KZ (Knizhnik-Zamolodchikov) equations when the Riemann surface varies, by showing that an analog of the rank-1 heat equation holds over these moduli spaces.

These results have not been rendered in terms of explicit functions except in genus 2, and in fact only in an extended sense (even-determinant case), nor have I been able to identify a source where the interpretation of all the mentioned aspects is given in a unified way.

In this talk, which is intended to be introductory and expository, I will detail the mentioned features and their interconnection; more precisely: (1) explicit Hamiltonians of the Hitchin system in genus 2, even-determinant case (joint work with B. van Geemen, and work by K. Gawędzky and P. Trang-Ngoc-Bich) including a 'strange duality' of projective-geometric nature; (2) interpretation of the integrals for the genus 2, odd-determinant case (W.M. Oxbury, unpublished D.Phil. thesis, Oxford, 1987); (3) geometric quantization in genus 2, even-determinant case (B. van Geemen and A. de Jong), as well as a(nother?) heat equation for hyperelliptic Riemann surfaces of any genus, even-determinant case; (4) KZ equations in genus 2 (K. Gawędzky and P. Trang-Ngoc-Bich); (5) ad hoc reduction of Hitchin to Neumann (rational-parameter Lax equations) in genus 2, even determinant (K. Gawędzky and P. Trang-Ngoc-Bich); (6) Lax representation for the Hitchin system in terms of Tyurin parameters (I.M. Krichever); (7) singular cases if there is time (N. Nekrasov; B. Enriques and V. Rubtsov).

This background will serve to formulate a current research program, articulated as follows: (1) description of $\mathcal{SU}_X(2,L)$ for X hyperelliptic, L even/odd (S. Ramanan, A. Beauville) and of $\mathcal{SU}_X(2,L)$ for X non-hyperelliptic of genus 3, L even (Coble); (2) construction of a geometric-quantization coordinate space in the cases given in (1), and of the class of functions for which the generalized heat equations are to be found.