On geodesic exponential maps of the Virasoro group

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Abstract

We study the geodesic exponential maps corresponding to Sobolev type right-invariant (weak) Riemannian metrics $\mu^{(k)}$ $(k \geq 0)$ on the Virasoro group and show that for $k \geq 2$, but *not* for $k = 0, 1$, each of them defines a smooth Fréchet chart of the identity. For $k = 0$ and $k = 1$ the corresponding geodesic flows are related to the Korteweg - de Vries and Camassa - Holm equations. In particular, the geodesic exponential map corresponding to the KdV equation $(k = 0)$ is *not* a local diffeomorphism near the origin.