Monodromy zeta-functions of deformations and Newton diagrams

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Abstract

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Since S. A. Broughton's paper on the topology of polynomial hypersurfaces there has been an enormous interest in the variation of topology of the fibers of a given polynomial, in particular due to its relation to the Jacobian Conjecture. For a level set $\{P_0 = c\}$ of the initial polynomial P_0 of a one-parameter deformation P_σ, its variation is the topology of the corresponding fibration over a punctured neighborhood of zero with the fiber $\{P_\sigma = c\}$ over a point σ. The zeta-function of the deformation of the fiber $\{P_0 = c\}$ is the zeta-function of the monodromy transformation of this fibration. S. Gusein-Zade and D. Siersma reduced the problem of computing this zeta-function to the one of computing "local" zeta-functions of deformations of germs.

In this paper we consider a germ of complex function F in $n+1$ variables at the origin. It defines a deformation f_σ of a germ of the complex function $f = f_0$. The deformation f_σ provides a fibration over a punctured neighborhood of the origin with the fiber $\{f_\sigma = 0\} \cap B_\varepsilon$ over a point σ, where $B_\varepsilon \subset \mathbb{C}^n$ is the closed ball of radius ε with the center at the origin. We provide a formula for the zeta-function $\zeta_{f_\sigma}(t)$ of this fibration in terms of the Newton diagram of F.

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