

BOSTON UNIVERSITY GEOMETRY SEMINAR

Sheaves and K-theory for \mathbb{F}_1 -schemes

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Wed., Dec. 1, 3-4 pm in PSY B53
Tea 2:45-3 in MCS 153

Abstract:

One of the problems motivating the development of a geometry over \mathbb{F}_1 , the "field with one element", is a reinterpretation of the stable homotopy of the sphere spectrum as K-theory of the "scheme" $\text{Spec } \mathbb{F}_1$.

This talk is based on a collaboration with Chenghao Chu and Rekha Santhanam. We will start with explaining why one expects the above reinterpretation. Then we review Deitmar's definition of an \mathbb{F}_1 -scheme (with a slight modification) and develop sheaf theory for these schemes. In order to define K-theory, one has to face two effects that do not occur in the theory of usual schemes: not every epimorphism of sheaves is a cokernel, and not every projective sheaf (in the categorical sense) is locally free. Once the definitions are set up in the right way, we can introduce K-theory for \mathbb{F}_1 -schemes via an adjusted version of Quillen's Q-construction. This realizes, in particular, the stable homotopy of the sphere spectrum as desired.