Moduli spaces of curves (Riemann surfaces) and Torelli Lie algebras. (After R. Hain.)

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Abstract

Let S be a compact orientable surface of genus g. The mapping class group Γ_g is the group of isotopy classes of orientation preserving diffeomorphisms of S. This group is the orbifold fundamental group of the moduli space of genus g Riemann surfaces.

The Torelli group T_g is the kernel of the natural homomorphism

$$\Gamma_g \to \operatorname{Aut} H_1(S, \mathbb{Z}).$$

It is known that T_g is finitely generated for $g \geq 3$, but it is not known for any $g \geq 3$ whether T_g is finitely presented.

However, one can say more on the Lie algebras level. Indeed, the Lie algebra of the unipotent completion of T_g is finitely presented for all $g \geq 3$, and moreover, all relations are quadratic when $g \geq 6$. This contains significant information about the group T_g. Along with other issues the proof uses the purity of the mixed Hodge structure on $H^2(\Gamma_g, V)$, where $g \geq 6$ and V is an irreducible representation of the algebraic group Sp_{2g}.