Harmonic tori and abelian integrals

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Abstract

For some time an outstanding conjecture in surface theory was that there could be no immersed surfaces of constant mean curvature in R^3 which were topologically different from the sphere. This conjecture was falsified by Wente in the mid 80's when he showed the existence of a CMC torus. Subsequent work showed that all such tori could be computed in terms of Riemann theta functions. The Gauss maps of such tori are non-conformal harmonic maps of a torus to S^2, and all such maps are Gauss maps of a (not necessarily compact) CMC surface. The aim of this talk is to present a purely geometric construction of each non-conformal harmonic torus on S^2 in terms of a "generalized Jacobi variety", which can be thought of as the parameter space for abelian integrals on a singular Riemann surface (or, if you prefer, the moduli space of line bundles of degree zero over this Riemann surface). Moreover, the construction generalizes easily to describe all non-isotropic harmonic tori in complex projective space of any dimension.