Vanishing theorems for the half-kernel of a Dirac operator

Maxim Braverman
Department of Mathematics
Ohio State University

Abstract

We obtain a vanishing theorem for the half-kernel of a Dirac operator on a Clifford module twisted by a sufficiently large power of a line bundle, whose curvature is non-degenerate at any point of the base manifold. In particular, if the base manifold is almost complex, we prove a vanishing theorem for the kernel of a Spin^c Dirac operator twisted by a line bundle with curvature of a mixed sign. In this case we also relax the assumption of non-degeneracy of the curvature. These results are generalization of a vanishing theorem of Borthwick and Uribe. As an application we obtain a new proof of the classical Andreotti-Grauert vanishing theorem for the cohomology of a compact complex manifold with values in the sheaf of holomorphic sections of a holomorphic vector bundle, twisted by a large power of a holomorphic line bundle with curvature of a mixed sign.

As another application we calculate the sign of the index of a signature operator twisted by a large power of a line bundle.