Verlinde formula and parabolic bundles

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Abstract:

The following is taken from the Mathematical Review of the speaker's paper *Espaces de modules de fibrs paraboliques et blocs conformes.* (French) [*Moduli spaces of parabolic bundles and conformal blocks*] in Duke Math. J. 84 (1996), no. 1, 217–235. This will be the topic of this talk.

This paper generalizes to parabolic bundles the Beauville-Laszlo proof of the Verlinde formulas [cf. A. Beauville and Y. Laszlo, Comm. Math. Phys. 164 (1994), no. 2, 385–419; MR 95k:14011]. G. Faltings' proof [J. Algebraic Geom. 3 (1994), no. 2, 347–374; MR 95j:14013], which holds for principal bundles of more general groups, uses a degeneration argument. The Beauville-Laszlo proof is very valuable as it introduces a rigorous algebro-geometric setting in infinite dimension. In addition to that, this paper develops the theory of the generalized theta divisor for parabolic bundles. In order to parametrize parabolic bundles over a given curve C (the extra data are a flag in the fibre E_x and corresponding parabolic weights for a finite set of points $x \in I \subset C$), the author considers the action of $\mathrm{SL}_r(A_C)$ on $\mathrm{SL}_r(C((z)))/\mathrm{SL}_r(C[\![z]\!]) \times \prod_{x \in I} \mathrm{Drap}_x$, where z is a local parameter at a point $p \in C$, A_C is the ring$H^0((C - \{p\}), \mathcal{O}_C)$ and Drap_x is the flag variety. He shows that the quotient under this action is isomorphic to the space of "algebraic fields" $\mathcal{PAR}$ (a suitable generalized moduli space, after G. Laumon and L. Moret-Bailly ["Champs algebriques", Preprint, Univ. Paris-Sud, Orsay, 1992; per bibl.]). In analogy with Beauville-Laszlo, he compares $\mathcal{PAR}$ with the Mehta-Seshadri moduli space $\mathcal{M}$, as well as the line bundles over these spaces. Lastly, he uses representation theory to assert that the space $H^0(\mathcal{M}, \Theta)$ equals $V_C^*(I, \bar{\lambda})$, the space of conformal blocks associated to a set $\{\lambda_x\}_{x \in I}$ of dominant weights of $\mathrm{sl}_r(A_C)$. This gives the Verlinde formula (a combinatorial expression for $\dim H^0(\mathcal{M}, \Theta)$) for this case, and specializes in

rank 2 to the one proved by A. Bertram [Invent. Math. 113 (1993), no. 2, 351–372; MR 95g:14011] via a geometric construction (complete extension spaces).

The author's style is remarkably clear and elegant.