Isotopy and diffeomorphisms in dimension 4

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Abstract
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Abstract: A natural question to ask about diffeomorphisms is whether they are isotopic, i.e. connected by a 1-parameter family of diffeomorphisms. There is a natural chain of implications:

Smooth isotopy $\Rightarrow$ topological isotopy $\Rightarrow$ homotopy.

Work of Quinn and Perron from the mid-80's says that homotopic homeomorphisms of simply-connected 4-manifolds are topologically isotopic, i.e. connected by a path of homeomorphisms. We will show that for 4-manifolds, the other arrow cannot be reversed, i.e. that there are homotopic diffeomorphisms which are not smoothly isotopic.

We will discuss a simple version of 1-parameter gauge theory which provides obstructions to smooth isotopy in dimension 4. Examples will be described for which this obstruction is non-zero. A more elaborate version, analogous to Donaldson's polynomials of 4-manifolds, can be used to prove the following.**Theorem:** *Let* $\mathrm{Diff}^h(Z)$ *denote the group of isotopy classes of diffeomorphisms of a manifold* Z *which are homotopic to the identity. There are manifolds for which* $\mathrm{Diff}^h(Z)$ *is infinitely generated.*