Gauss diagram formulas for Vassiliev invariants

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Abstract

Integral formulas of Bar-Natan and Kontsevich for the Vassiliev knot invariants are higher-degree generalizations of the Gauss integral. It is however well-known that the Gauss integral for the linking number of a 2-component link has a simple combinatorial counter-part. Namely, the linking number may be computed by counting the signs of double points of a link diagram.

We establish a similar technique for higher degree Vassiliev invariants and show that they can be computed from a link diagram by counting the products of signs over appropriate configurations of double points. This leads to elementary combinatorial expressions for the invariants.

A geometry underlying this combinatorial approach is discussed. We show that, similarly to the case of a linking number, any Vassiliev invariant may be interpreted as the degree of a map. This interpretation leads to a unification of various kinds of formulas. We consider in details the model example of the Casson knot invariant v_2.