Gradient-Like Flows and Self-Indexing in Stratified Morse Theory

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Abstract

A self-indexing Morse function f on a smooth manifold is a function whose index at every critical point p is equal to f(p). The existence of self-indexing functions was first used by Smale in his proof of the Poincare Conjecture in dimensions ¿ 4. In this talk we will discuss the existence of self-indexing (real-valued) functions on a smooth complex algebraic variety X with a fixed algebraic stratification. Here the notions of a Morse function, a critical point, and the index must all be understood in the stratified sense. To show that X admits many self-indexing functions, we examine the ascending and descending sets for a suitable gradient-like vector field in the neighborhood of a critical point.

The motivation for this work is the theory of (middle perversity) perverse sheaves on X. We will indicate some applications to this theory.